# Inspect
# CCR Performance Tasks

## Geometry: Visualize Relationships Between 2-D and 3-D Shapes

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## Inspect offers the following assessment products:

| | |
|---|---|
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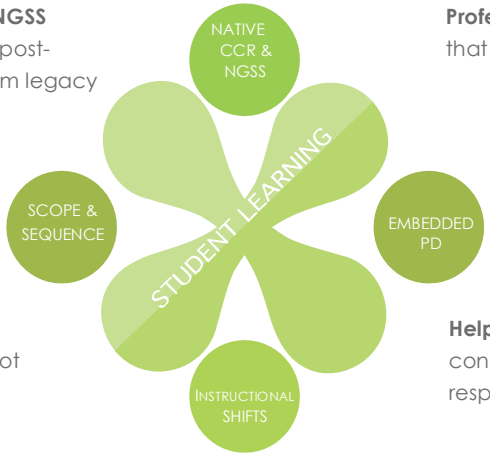
Inspect Assessment Content is available through a variety of assessment administration and data analysis platforms.

## Inspect assessment content offers these benefits:

**Native college- and career-ready and NGSS content** prepares students to meet their post-secondary goals. Content re-aligned from legacy standards cannot do this.




**Professional development embedded** within content that

- shows the relationship between specific skills and higher-order thinking
- includes authentic, permissioned texts of appropriate complexity
- and documents student progress using DOK and learning progressions

**Content that addresses your scope and sequence** so that your assessments do not waste valuable instruction time

**Help for teachers addressing the instructional shifts** with content that elicits evidence of learning from each response

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# CCR Performance Tasks

## Geometry: Visualize Relationships Between 2-D and 3-D Shapes

Student Test Booklet

**Name:**

# Geometry: Visualize Relationships Between 2-D and 3-D Shapes

**Student Rubric**

This problem is meant to test if you can:
- Visualize the various cross sections of shapes.
- Precisely and accurately use words, diagrams, or drawings to communicate the shapes that result from cutting a three-dimensional figure with a plane.

Your teacher will rate your answer as a level 4, 3, 2, 1, or 0. The descriptions below explain the types of answers expected at each level.

**Level 4:**
Your answer is correct and complete. Your answer includes:
- Sketches of three different cross-sectional shapes that are generated by cutting a cube with a plane, including one complex cross section;
- Sketches and an accurate, written description of two different three-dimensional figures from which a solid shape was removed when cut by a plane.

**Level 3:**
Your answer is correct but you have made minor errors. Your answer includes:
- Sketches of three different cross-sectional shapes that are generated by cutting a cube with a plane, with minor errors on more complex cross sections;
- Sketches and an accurate, written description of two different three-dimensional figures from which a solid shape was removed when cut by a plane, with only minor errors.

**Level 2:**
You have answered part of the problem correctly but some of your sketches and descriptions are missing or weak. Your answer includes:
- Sketches of two or three different cross-sectional shapes that are generated by cutting a cube with a plane. Only basic cross sections are correct. The more complex cross sections have errors;
- Sketches and accurate, written descriptions of one or two three-dimensional figures from which a solid shape was removed when cut by a plane, although the sketches and descriptions contain errors.

**Level 1:**
Your answers are incorrect. Your answer includes:
- A sketch of one or more cross-sectional shapes that are generated by cutting a cube with a plane, with minor or major errors on all shapes;
- Incorrect sketches and an inaccurate, written description of a three-dimensional figure from which a solid shape was removed when cut by a plane.

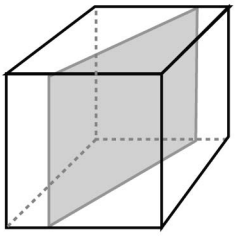
**Level 0:**
Your answer is not related to the question, the teacher cannot understand your answer, or you do not write anything.

**Name:**_____

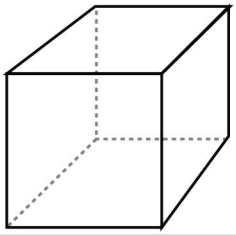
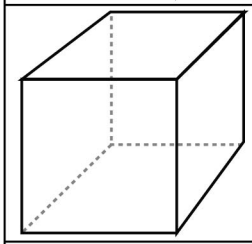
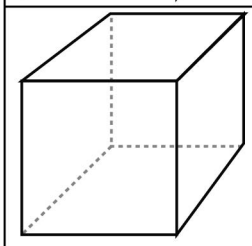
# Geometry: Visualize Relationships Between 2-D and 3-D Shapes

Complete all the tasks in the test booklet.

**1** **A. A cube can be cut by a plane in many different ways, resulting in new faces with many different shapes. For example, if a cube is cut by a plane through its center and perpendicular to its base, the cross-sectional shape is a rectangle, as shown below. If the cut is made perpendicular to the base and to the sides, the cross-sectional shape is a square.**



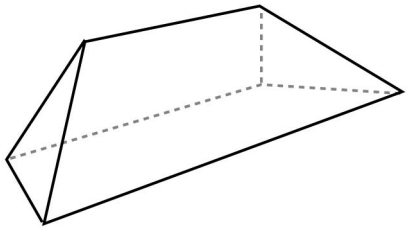
**In the table below, sketch three other cross-sectional shapes that are possible when a cube is cut by a plane. Show how each plane cuts through the cube on the left, and then sketch the two-dimensional cross section that is formed by the cut.**

| Cut Made by Plane | Two-Dimensional Cross Section |
|---|---|
|  | |
|  | |
|  | |

**Name:**_____

# Visualize Relationships Between 2-D and 3-D Shapes

**B. The shape shown below was removed from a three-dimensional shape when it was cut by a plane. The shape has five faces: three of the faces are trapezoids, and the other two faces are triangles.**



**Sketch at least two three-dimensional figures from which the shape could have been removed, and show where the plane cut each figure. In addition, provide a verbal description explaining the types of figures that yield such a shape when cut by a plane.**

# CCR Performance Tasks

## Geometry: Visualize Relationships Between 2-D and 3-D Shapes

Teacher Guide

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# About the Teacher Guide

This document contains support materials for “Geometry: Visualize Relationships Between 2-D and 3-D Shapes.”
This includes:

(a) The task
(b) The standards and depth of knowledge level of the task
(c) The scoring rubric
(d) Discussion questions
(e) Extension activities

These specifications have been included to help you connect the task to the Common Core content standards and the standards for mathematical practice. The rubric is designed to help you look for the development of mathematical practices in student work. It is also here to help you look for consistencies in student content errors that can help guide intervention and reteach strategies.

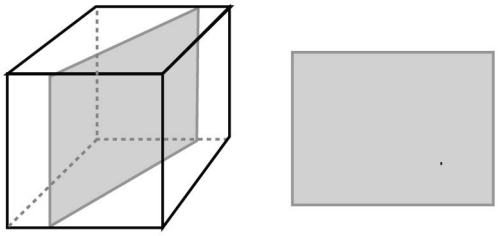
**Test Definition File**

| Item # | Correct Answer | Practice Standard | Content Standards |
|---|---|---|---|
| 1 | See Scoring Rubric | Mathematical Practice 6 | G-GMD.4 |

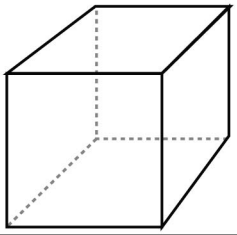
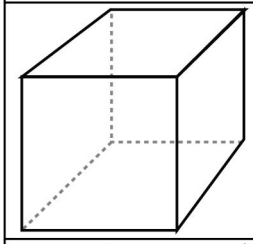
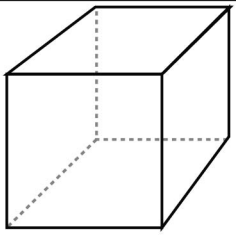
| SBAC Claims | PARCC Sub-Claims |
|---|---|
| 1 and 2 | A and C |

## Performance Task

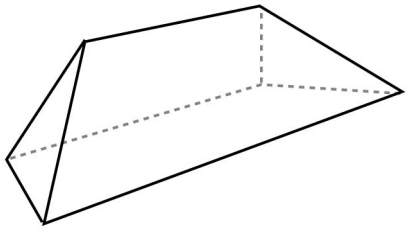
**A. A cube can be cut by a plane in many different ways, resulting in new faces with many different shapes. For example, if a cube is cut by a plane through its center and perpendicular to its base, the cross-sectional shape is a rectangle, as shown below. If the cut is made perpendicular to the base and to the sides, the cross-sectional shape is a square.**



**In the table below, sketch three other cross-sectional shapes that are possible when a cube is cut by a plane. Show how each plane cuts through the cube on the left, and then sketch the two-dimensional cross section that is formed by the cut.**

| Cut Made by Plane | Two-Dimensional Cross Section |
|---|---|
|  | |
|  | |
|  | |

**B. The shape shown below was removed from a three-dimensional shape when it was cut by a plane. The shape has five faces: three of the faces are trapezoids, and the other two faces are triangles.**



**Sketch at least two three-dimensional figures from which the shape could have been removed, and show where the plane cut each figure. In addition, provide a verbal description explaining the types of figures that yield such a shape when cut by a plane.**

# Standards Alignment

## Practice Standards

### MP6 > DOK 3

Attend to precision -- Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

## Content Standard

### G-GMD.4

Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

## SBAC Claim

**Mathematics Claim #1:**
Concepts and Procedures. Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency.

**Mathematics Claim #2:**
Problem Solving. Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem solving strategies.

## PARCC Sub-Claims

**Sub-Claim A:**
Major Content with Connections to Practices. The student solves problems involving the Major Content for her grade/course with connections to the Standards for Mathematical Practice.

**Sub-Claim C:**
Highlighted Practices MP.3, 6 with Connections to Content: expressing mathematical reasoning. The student expresses grade/course-level appropriate mathematical reasoning by constructing viable arguments, critiquing the reasoning of others, and/or attending to precision when making mathematical statements.

# Scoring Rubric

**4 Point Response:**
The response demonstrates a high level of understanding, including:

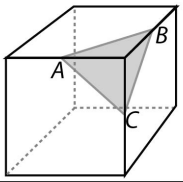
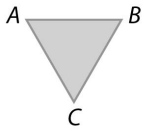
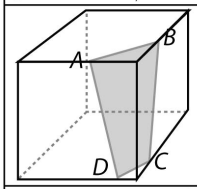
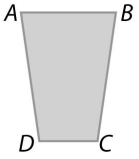
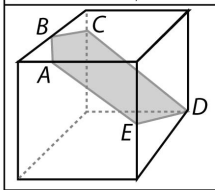
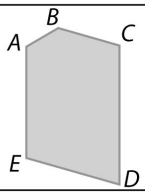
- The ability to visualize and to produce accurate graphical representations of the possible two-dimensional cross sections of a three-dimensional object;
- The ability to visualize and accurately represent and describe a three-dimensional object containing a particular cross section.

A level 4 response is characterized by:

- Sketches of three different cross-sectional shapes that are generated by slicing a cube with a plane;
- Accurate sketches and text descriptions of two different three-dimensional figures from which a solid shape could have been removed when sliced by a plane.

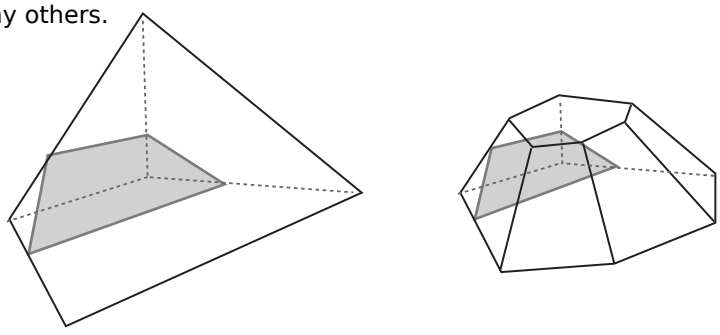
A sample level 4 response follows.

Part A: In addition to the rectangle described in the problem, cross-sectional faces in the shape of triangles, pentagons, and hexagons can also be formed by slicing the cube with a plane, as shown below.

| Cut Made by Plane | Two-Dimensional Cross Section |
|---|---|
|  |  |
|  |  |
|  |  |

Part B:

The trapezoidal pyramid and the hexagonal frustum shown below are two figures that yield the removed piece, though there are many others.



"These two shapes can be cut by a plane to yield the given piece. Any three-dimensional figure in which four faces of the figure have the same interfacial angles as the removed piece can yield that piece, with the fifth face being produced by the plane that cuts the figure."

**3 Point Response:**
The response demonstrates a strong understanding, but the work contains minor errors. A level 3 response is characterized by:

- The ability to visualize and graphically represent three possible two-dimensional cross sections of a three-dimensional object, though one graphical representation may be a complex cross section and contain errors;
- The ability to visualize, represent, and describe two examples of three-dimensional objects containing a particular cross section, though the graphic representation or verbal explanation may be incomplete.

**2 Point Response:**
The response demonstrates a basic but incomplete understanding. A level 2 response is characterized by:

- A basic ability to visualize and graphically represent the possible two-dimensional cross sections of a three-dimensional object, with one simplistic cross section that is correct and with sketches of one or two more complex cross sections that contain errors;
- A basic ability to visualize, represent, and describe a three-dimensional object containing a particular cross section, though the verbal explanation may be incomplete or include minor errors, and the answer may include only one figure.

**1 Point Response:**
The response demonstrates minimal understanding. A level 1 response is characterized by:

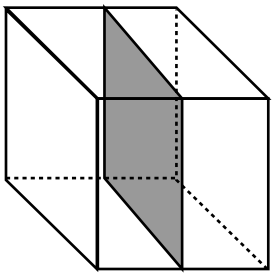
- A weak ability to visualize two-dimensional cross sections of a three-dimensional object, with sketches of one possible cross section, with major errors;
- A weak ability to visualize a three-dimensional object containing a particular cross section, with an incorrect sketch and verbal explanation of only one possible figure.

**0 Point Response:**
There is no response, or the response is off topic.

# Discussion Questions

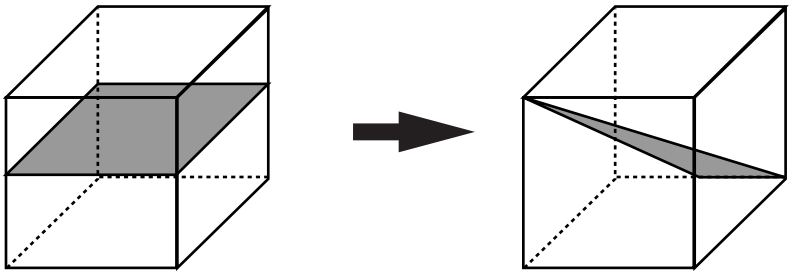
**Use the following questions to help students struggling to access the problem. Encourage students to sketch the cross sections or use a 3-D graphing program to help them visualize how the cross sections change.**

1. If a cube is cut by a plane that is perpendicular to the bottom and top of the cube, you get a rectangle. How does the rectangle change as you rotate the plane from parallel to the left and right faces of the cube to perpendicular to the left and right faces of the cube? When does the cross section have the greatest length?



**Possible response:** *The cross section is a square like the faces of the cube when the plane is parallel to the sides of the cube and perpendicular to the bottom and top of the cube. As you rotate it, the length increases until the plane cuts diagonally across the top and bottom of the cube. This is the longest cross section. As you continue to rotate the cube, the length decreases until you are back to a square.*

2. The picture shows a cube cut by a plane that is parallel to the top and bottom of the cube. What happens to the cross section as you tilt the plane so that it cuts from the top-front corner to the bottom-rear corner of the cube, as shown?



**Possible response:** *As you tilt the plane, the front-left corner and bottom-right corner become smaller angles while the corners that stay in the middle of the opposite edges become larger angles. The side lengths become a bit longer as well, but all the side lengths stretch at the same rate. The cross section becomes a rhombus.*

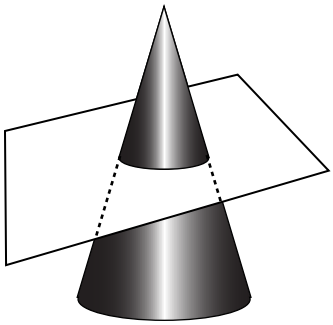
3. What happens to the cross section if you tilt the plane beyond the front-upper-left corner and rear-lower-right corner so the plane cuts through the top face of the cube and the bottom face of the cube?

**Possible response:** *The smaller angles of the rhombus are cut off so you get a six-sided figure. The obtuse angles of the original rhombus continue to get more obtuse until the plane is perpendicular to the top and bottom faces of the cube. At that point, the cross section is a rectangle again because the obtuse angles have become straight angles.*

## Extension Activities

1. Explore cross sections that can result from cutting shapes such as cones.

   A. What are the possible cross sections that result when a plane cuts through a cone?



   **Sample response:** *Two possible cross sections are circles and ellipses. However, if the angle of the plane is changed, the cross section can also be a parabola or a hyperbola. If the plane cuts exactly through the tip of the cone and the base, the cross section is a triangle.*

2. Given a particular cross section, find the shapes from which the cross section can have come.

   A. Is it possible to obtain an octagonal cross section from a cube?

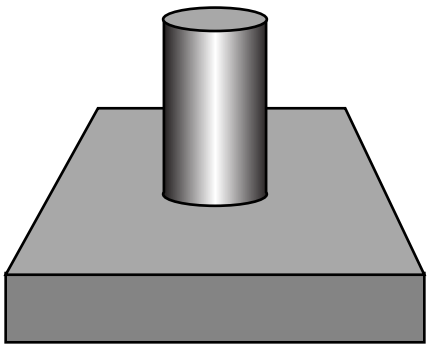
   **Sample response:** *It is not possible to obtain an octagonal cross section from a cube, because a cube has only six faces. Six is the maximum number of sides that a cross section can contain.*

   B. What kinds of three-dimensional figures can have cross sections in the shape of an octagon?

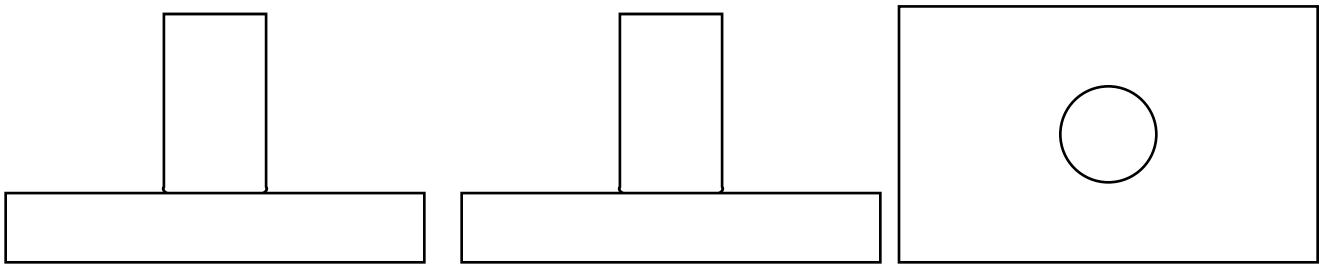
   **Sample response:** *Any shape that has at least eight faces can have a cross section in the shape of an octagon, assuming that a plane can cut through each face. One example is an octagonal prism.*

3. Work with composite three-dimensional shapes to visualize different views of these shapes and to determine a shape that has a particular set of views.

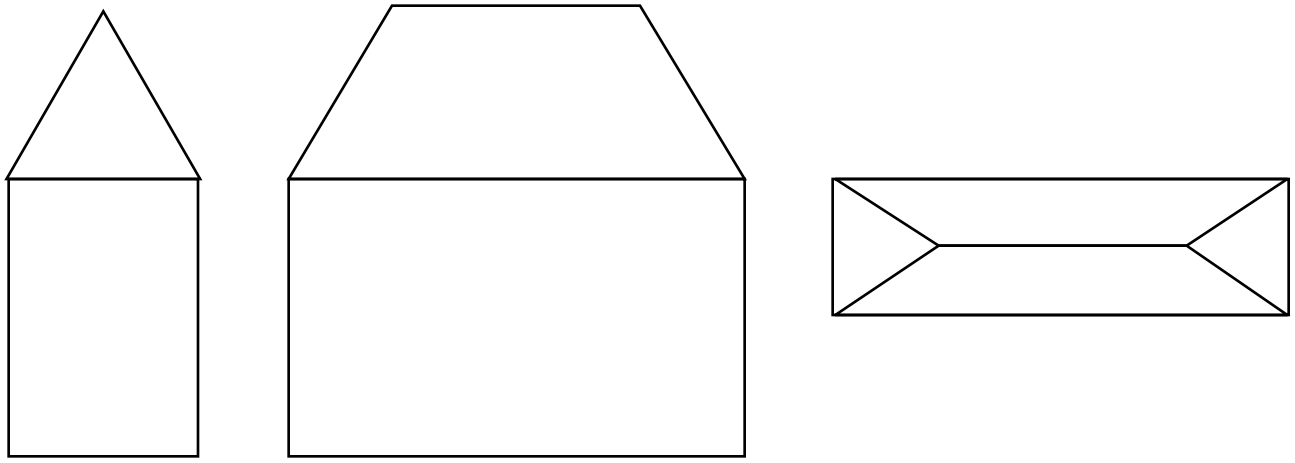
   A. Draw the front, side, and top views of the object shown below.



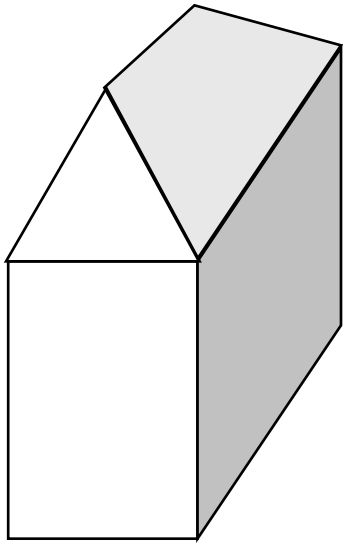
**Sample response:** *The front and side views are the same, but the top view is a rectangle with a circle in it.*



B. The image below shows the side, front, and top view of a three-dimensional object. Sketch the object in three dimensions.



**Sample response:**



4. Use three-dimensional coordinate geometry to calculate side lengths and angle measures of cross sections. This allows you to check the accuracy of your sketches of complex cross sections.

Consider a cube in three-dimensional space.
- Place the rear-left corner at the origin and make each side length 4 units.
- Sketch a cross section you want to analyze.
- Assign coordinates to three vertices of the cross section.
- Determine the equation of the plane that contains those three points.
- Use the equation of the plane to determine unknown coordinates of other vertices of the cross section.
- Use the distance formula to find the lengths of edges.
- Use the law of cosines to find the unknown angle measures.
- Sketch the two-dimensional view of the cross section using the side lengths and angle measures you determined.