Inspect
CCR Performance Tasks

Algebra II: Extended Performance Task
Designing the Perfect Package

Inspect offers the following assessment products:

Content Bank for English/Language Arts and Math Grades 2 – High School	▪ More than 36,000 items ▪ More 1500 complex texts, including authentic permissioned texts ▪ Includes Literacy in History, Social Science, Science, and Technical Subjects
Quick Checks for English/Language Arts and Math Grades 2 – High School	▪ Fixed-form assessments with five to seven items including constructed response ▪ Key instructional concepts embedded in standards (clusters for Math, staircase of text complexity for ELA)
Focused Interim Assessments for English/Language Arts and Math Grades 3 – High School	▪ Prebuilt assessments with up to 15 items that focus on groups of related standards within a Claim or domain ▪ More focused than summative assessments ▪ Flexible and customizable ▪ Mirrors SBAC IAB blueprints
NGSS Formative Assessments Grades 5 – High School	▪ Prebuilt assessments with items linked to experimental contexts that assess the three dimensions of science learning ▪ Flexible and customizable ▪ Addresses the California Course Models and NGSS Bundles
Observational Tasks for English/Language Arts and Math Grades K - 1	▪ Developmentally appropriate for individual students and small groups

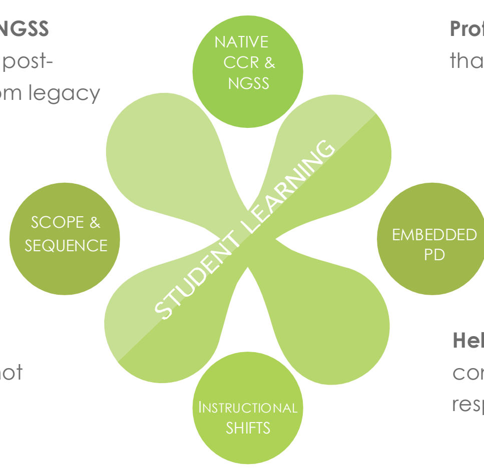
Inspect Assessment Content is available through a variety of assessment administration and data analysis platforms.

Inspect assessment content offers these benefits:

Native college- and career-ready and NGSS content prepares students to meet their post-secondary goals. Content re-aligned from legacy standards cannot do this.

Professional development embedded within content that

- shows the relationship between specific skills and higher-order thinking
- includes authentic, permissioned texts of appropriate complexity
- and documents student progress using DOK and learning progressions



Content that addresses your scope and sequence so that your assessments do not waste valuable instruction time

Help for teachers addressing the instructional shifts with content that elicits evidence of learning from each response

We constantly update our content. Ask us about what's new!

info@illuminateed.com **www.illuminateed.com**

CCR Performance Tasks

Algebra II: Extended Performance Task
Designing the Perfect Package

Student Test Booklet

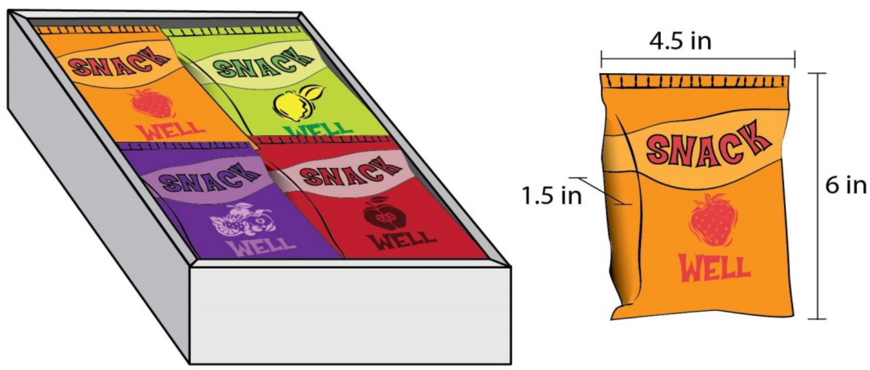
Name:

Name:________________________

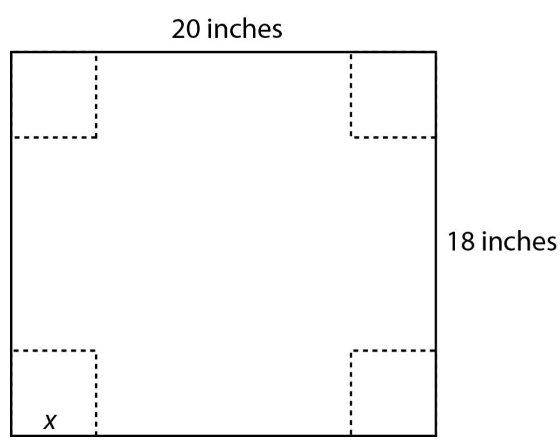
Algebra II: Extended Performance Task: Designing the Perfect Package

Complete all the tasks in the test booklet.

After graduating from school, you take a summer job with a local company that manufactures and packages bagged snack food. Your first project is to make sure the current package is the best fit for the product and the most cost efficient. Each package contains 8 snack bags. After you complete your research, you will need to compose a comprehensive report detailing your findings and suggested packaging size. Your report should be based on your work with equations and the analysis of the equations along with visual graphs showing key features. These equations and graphs will be instrumental in your decision on the ideal package to be constructed from the same-size cardboard the company currently uses.



The company is currently using a 20 in. by 18 in. piece of cardboard to make the packaging for the snack bags. The first step in creating the packaging is to cut out 4 equal squares to create an open-top box. Your project manager gives you a copy of the diagram they use before cutting out the packaging. You must figure out the dimensions of the 4 equal squares. The sides are folded up to form a rectangular prism. The top of the box is left open to display the snack bags that are being sold. A piece of thick, clear plastic is stretched across the top and around the package to seal the snack bags inside the box.



Name:________________________

Algebra II: Extended Performance Task: Designing the Perfect Package

Part A: Dimensions of the Current Package

1. Using the diagram on the previous page, write algebraic expressions to represent the three dimensions, in inches, of a package built using the company's process with the current cardboard piece. Explain how you determined the dimensions.

2. Represent the constraints on the possible values of *x* for a package. Explain how you found these constraints.

Name:_________________________

Algebra II: Extended Performance Task: Designing the Perfect Package

Optional Support Worksheet for Questions 1 and 2

1. Using the diagram given on the first page, write expressions in terms of x to represent the three dimensions, in inches, of a package built using the company's process. Label each expression with its dimensions.

Explain the process you used to determine the expressions that represent each dimension of the package.

Name:________________________

Algebra II: Extended Performance Task: Designing the Perfect Package

Not all real values of x are reasonable given the context of the problem.

2. Write and solve an inequality to represent the constraints on x for each of the three dimensions of a package. Label each expression with its dimensions. Show your work.

Combine these inequalities into a compound inequality that represents the possible values of x for a package. (Use the number lines below the response box to help visualize the overlap of each inequality.)

What does the compound inequality mean in terms of the context?

Name:________________________

Algebra II: Extended Performance Task: Designing the Perfect Package

3. The surface area of the top of the current package (the display portion) is 120 in^2. What are the dimensions of the square being cut out? Show your work.

4. What is the volume of the package? Show all of your work.

Optional Support Worksheet for Questions 3 and 4

3. The surface area of the top of the current package (the display portion) is 120 in^2. Using this surface area, write an equation using the expressions from #1 and then solve for *x*.

What are the dimensions of the square being cut out? Explain your answer in terms of the context.

4. What are the dimensions of the package that will be used to find the volume? What is the volume of the package? Show all of your work.

Name:_________________________

Part B (optional): Group Activity

Construct a basic package similar to the one in this task. You will need a piece of paper, ruler, scissors, and tape in order to complete the package. Each person in your group should make their own package.

What are the dimensions of the squares that you are to remove from each corner?

____

Cut out four congruent squares from each corner of the rectangle. Fold the sides up to form a rectangular prism and tape the corners.

Answer the following questions about the package you constructed.

What is the volume of your package? How did you find the volume?
What is the surface area of the display portion of your package?
How is your package different from the others in your group?

Discuss with your group some limitations on constructing a package using this process.

What are some of the limitations that your group found? List them in the response box below.

How do you think different sizes of packaging could affect sales of the product? Discuss this with your group and then write some of the answers in the response box below.

Part C: Maximizing the Volume

The company wants to see whether cutting a different-sized square from each corner of a piece of cardboard with the same dimensions could increase the volume of the package. The company pays $0.005 per square inch for the uncut cardboard pieces.

5. Write a function that represents the volume of a package. Show your work.

6. Graph the function on the coordinate grid below that represents the volume of a package depending on *x*, the length of each side of the square being cut out of the corners of the rectangular piece of cardboard.

Name:________________________

Algebra II: Extended Performance Task: Designing the Perfect Package

Optional Support Worksheet for Questions 5 and 6

5. Write a function that represents the volume of a package in terms of *x*. Write your final answer in standard polynomial form.

6. Graph the function on the coordinate grid that represents the volume of a package depending on *x*, the length of each side of the square being cut out of the corners of the rectangular piece of cardboard.

What are the key features of the graph (*x*-intercept, *y*-intercept, maximum or minimum point, and end behavior)?

Name:________________________

Algebra II: Extended Performance Task: Designing the Perfect Package

Name:_________________________

7. What do the *x*-intercepts of this graph represent? Explain what each interval of the graph represents. Make sure to explain how each interval relates to the context of the problem.

Optional support worksheet for Question 7

7. What do the *x*-intercepts of this graph represent?

Explain what each interval of the graph represents. Make sure to explain how each interval relates to the context of the problem.

Interval #1:

Interval #2:

Interval #3:

Interval #4:

8. What is the maximum volume of a package? What would be the dimensions of the square cutout? Show all of your work.

9. How does the volume of the current package compare to the possible maximum volume of a package? Explain if you think it is a good idea to switch from the original dimensions to these new dimensions.

Name:__________________________

Optional Support Worksheet for Questions 8 and 9

8. What is the maximum volume of a package? How do you know this is the maximum volume?

What would be the dimensions of the square cutout that would produce a package with the maximum volume? How are these dimensions different from the current square cutout? Do any of the other dimensions change? If so, explain.

9. How does the volume of the current package compare to the possible maximum volume of a package? What is the difference between the two volumes?

The cost of each cardboard may be a factor in making your decision on what the ideal dimensions for your company's packaging should be.

10. How much money does each cardboard cost the company? Show all of your work.

Using the information you found in this part of the task, give a complete answer to the following question.

11. What would you suggest the company use as the dimensions of the package so that it is the most cost efficient: the current package dimensions, the dimensions for the maximum volume, or another set of dimensions? Explain your answer.

Name:__________________________

Algebra II: Extended Performance Task: Designing the Perfect Package

Optional Support Worksheet for Questions 10 and 11

10. How much money does each cardboard cost the company? Show all of your work.

How much money is wasted when the square cutouts are discarded before the package is assembled? Show all of your work.

11. How much money is wasted when the square cutouts are discarded before the package is assembled if the dimensions that produce the maximum volume are used? Show all of your work.

Name:_______________________

Algebra II: Extended Performance Task: Designing the Perfect Package

Is there another set of dimensions that can be used to generate a package from the cardboard? If so, how much money is wasted when the square cutouts are discarded before the package is assembled when using this new set of dimensions? Show all of your work.

What would you suggest the company use as the dimensions of the package so that it is the most cost efficient: the current package dimensions, the dimensions for the maximum volume, or another set of dimensions? Explain your answer.

Name:_______________________

Part D: The Perfect Package Plan

12. Based on the results of this project, would you keep the package at its current dimensions? If not, what size of a square cutout would you recommend the company use for its new package? What are your suggestions to the company to ensure they are using the ideal packaging for their product? Write a paragraph with at least 8 sentences explaining your answer using examples from all parts of this task.

illuminate education

CCR Performance Tasks

Algebra II: Extended Performance Task
Designing the Perfect Package

Teacher Guide

Task Specifications

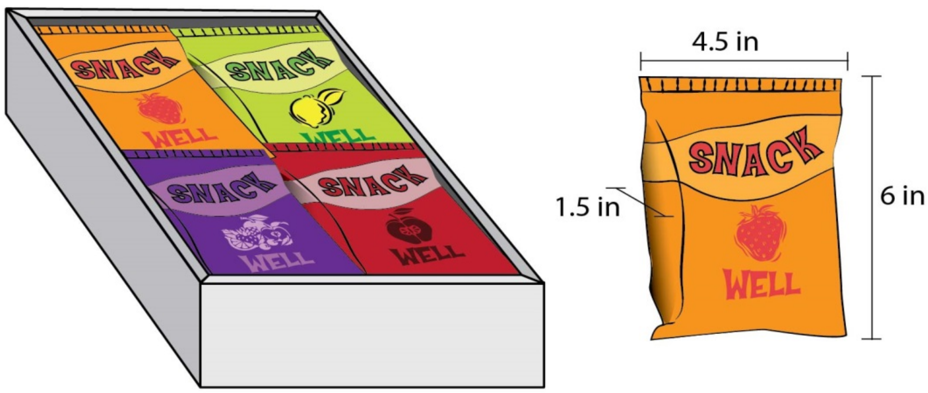
Content Area	Mathematics
Title	Designing the Perfect Package
Grade Level	Algebra II
Problem Type	Extended Performance Task
Standards for Mathematical Practices	**Mathematical Practice 1 (MP.1):** Make sense of problems and persevere in solving them. Mathematically proficient students: • Explain to themselves the meaning of a problem and look for entry points to its solution. • Analyze givens, constraints, relationships, and goals. • Make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. • Consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. • Monitor and evaluate their progress and change course if necessary. • Explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. • Check their answers to problems using a different method, and continually ask themselves, “Does this make sense?” • Understand the approaches of others to solving complex problems and identify correspondences between different approaches. **Mathematical Practice 4 (MP.4):** Model with mathematics. Mathematically proficient students: • Solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. • Can apply what they know and are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. • Identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas and analyze these relationships mathematically to draw conclusions. • Interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.
Common Core State Standards	**A.SSE.1a** Interpret expressions that represent a quantity in terms of its context. • Interpret parts of an expression, such as terms, factors, and coefficients. **A.SSE.3a** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. • Factor a quadratic expression to reveal the zeros of the

	function it defines. **A.SSE.3b** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression • Complete the square in a quadratic expression to reveal the maximum value of the function it defines. **A.CED.1** Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* **A.CED.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.* **A.REI.3** Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. **A.REI.4b** Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b. **F.BF.1** Write a function that describes a relationship between two quantities. **F.IF.3** Use the process of completing the square to show extreme values and interpret these in terms of a context. **F.IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features, given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. **F.IF.7a** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. • Graph linear and quadratic functions and show intercepts, maxima, and minima. **F.IF.7c** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. • Graph polynomial functions, identifying zeros and showing end behavior. **F.IF.8** Analyze functions using different representations. **F.IF.9** Compare properties of two functions, each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the greater maximum.*
CCSS Literacy in Writing-Grade 9-10	**W.11-12.2.a** Write informative/explanatory texts to examine and convey complex ideas, concepts, and information clearly and accurately through the effective selection, organization, and analysis of content.

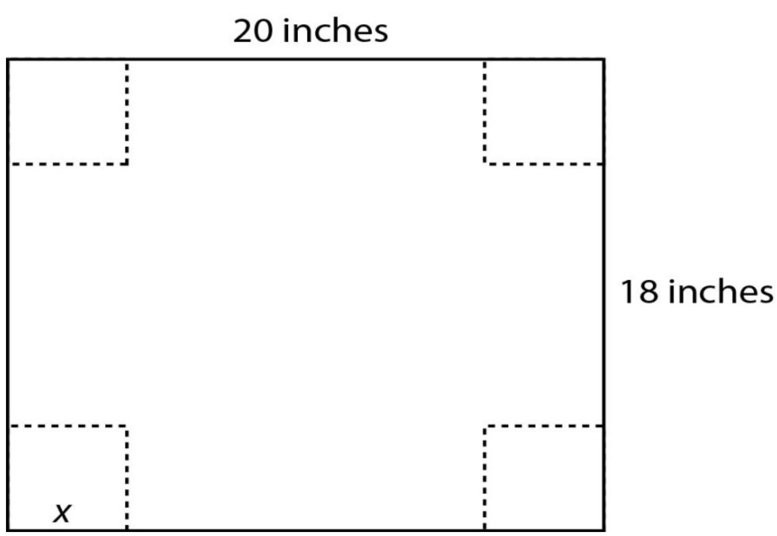
	a. Introduce a topic; organize complex ideas, concepts, and information so that each new element builds on that which precedes it to create a unified whole; include formatting (e.g., headings), graphics (e.g., figures, tables), and multimedia when useful to aiding comprehension. **W.11-12.2.b** Write informative/explanatory texts to examine and convey complex ideas, concepts, and information clearly and accurately through the effective selection, organization, and analysis of content. b. Develop the topic thoroughly by selecting the most significant and relevant facts, extended definitions, concrete details, quotations, or other information and examples appropriate to the audience's knowledge of the topic.
SBAC Assessment Claims	**Claim 2: Problem Solving**—Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem solving strategies.
PARCC Assessment Claims	**Sub-Claim D: Highlighted Practice MP.4 with Connections to Content (modeling/application)—**The student solves real-world problems with a degree of difficulty appropriate to the grade/course by applying knowledge and skills articulated in the standards for the current grade/course (or for more complex problems, knowledge and skills articulated in the standards for previous grades/courses), engaging particularly in the Modeling practice.
Depth of Knowledge	**Level 4: Extended Strategic Thinking**—Curricular elements assigned to this level demand extended use of higher order thinking processes such as synthesis, reflection, assessment and adjustment of plans over time. Students are engaged in conducting investigations to solve real-world problems with unpredictable outcomes. Employing and sustaining strategic thinking processes over a longer period of time to solve the problem is a key feature of curricular objectives that are assigned to this level. Key strategic thinking processes that denote this particular level include: synthesize, reflect, conduct, and manage.
Task Purpose	In this task you will be asked to figure out the dimensions of a package that a company currently uses to display one of its products. You will develop equations and analyze the equations' graphs and key features in order to determine the maximum volume possible for a package constructed from the same-size cardboard as the company currently uses. Using this information, you will decide whether the company's current package is ideal or whether the company should make modifications to the dimensions of the package being used. All of your evidence will be detailed in a written report. The purpose of this task is to assess students' ability to write, graph, and interpret quadratic and polynomial functions in order to solve problems arising in the workplace.

Student Task

After graduating from school, you take a summer job with a local company that manufactures and packages bagged snack food. Your first project is to make sure the current package is the best fit for the product and the most cost efficient. Each package contains 8 snack bags. After you complete your research, you will need to compose a comprehensive report detailing your findings and suggested packaging size. Your report should be based on your work with equations and the analysis of the equations along with visual graphs showing key features. These equations and graphs will be instrumental in your decision on the ideal package to be constructed from the same-size cardboard the company currently uses.



The company is currently using a 20 in. by 18 in. piece of cardboard to make the packaging for the snack bags. The first step in creating the packaging is to cut out 4 equal squares to create an open-top box. Your project manager gives you a copy of the diagram they use before cutting out the packaging. You must figure out the dimensions of the 4 equal squares. The sides are folded up to form a rectangular prism. The top of the box is left open to display the snack bags that are being sold. A piece of thick, clear plastic is stretched across the top and around the package to seal the snack bags inside the box.



Part A: Dimensions of the Current Package

1. Using the diagram on the previous page, write algebraic expressions to represent the three dimensions, in inches, of a package built using the company's process with the current cardboard piece. Explain how you determined the dimensions.

2. Represent the constraints on the possible values of *x* for a package. Explain how you found these constraints.

3. The surface area of the top of the current package (the display portion) is 120 in^2. What are the dimensions of the square being cut out? Show your work.

4. What is the volume of the package? Show all of your work.

Part B (optional): Group Activity

Construct a basic package similar to the one in this task. You will need a piece of paper, ruler, scissors, and tape in order to complete the package. Each person in your group should make their own package.

What are the dimensions of the squares that you are to remove from each corner? _______________

Cut out four congruent squares from each corner of the rectangle. Fold the sides up to form a rectangular prism and tape the corners.

Answer the following questions about the package you constructed.

What is the volume of your package? How did you find the volume?
What is the surface area of the display portion of your package?
How is your package different from the others in your group?

Discuss with your group some limitations on constructing a package using this process.

What are some of the limitations that your group found? List them in the response box below.

How do you think different sizes of packaging could affect sales of the product? Discuss this with your group and then write some of the answers in the response box below.

Part C: Maximizing the Volume

The company wants to see whether cutting a different-sized square from each corner of a piece of cardboard with the same dimensions could increase the volume of the package. The company pays $0.005 per square inch for the uncut cardboard pieces.

5. Write a function that represents the volume of a package. Show your work.

6. Graph the function on the coordinate grid below that represents the volume of a package depending on *x*, the length of each side of the square being cut out of the corners of the rectangular piece of cardboard.

7. What do the *x*-intercepts of this graph represent? Explain what each interval of the graph represents. Make sure to explain how each interval relates to the context of the problem.

8. What is the maximum volume of a package? What would be the dimensions of the square cutout? Show all of your work.

9. How does the volume of the current package compare to the possible maximum volume of a package? Explain if you think it is a good idea to switch from the original dimensions to these new dimensions.

The cost of each cardboard may be a factor in making your decision on what the ideal dimensions for your company's packaging should be.

10. How much money does each cardboard cost the company? Show all of your work.

Using the information you found in this part of the task, give a complete answer to the following question.

11. What would you suggest the company use as the dimensions of the package so that it is the most cost efficient: the current package dimensions, the dimensions for the maximum volume, or another set of dimensions? Explain your answer.

Part D: The Perfect Package Plan

12. Based on the results of this project, would you keep the package at its current dimensions? If not, what size of a square cutout would you recommend the company use for its new package? What are your suggestions to the company to ensure they are using the ideal packaging for their product? Write a paragraph with at least 8 sentences explaining your answer using examples from all parts of this task.

Teacher Instructions

This performance task is designed to assess student understanding of a variety of standards and claims. The task was designed with the understanding that all classrooms are different. Some groups may need extension activities, some may need to reduce the number of days planned for this task, and some may need to omit or simplify certain parts, such as those involving third degree polynomials and their graphs, depending on what time during the school year this task is given. As such, recommended extension activities and adjustments are included here and in teacher notes throughout the task.

Test Definition File

Item	Correct Answer	Practice Standard	Common Core Standards
1	See Scoring Rubric	Mathematical Practice 2 and 4	A.SSE.1a, A.SSE.3a, A.SSE.3b, A.CED.1, A.CED.3, A.REI.3, A.REI.4b, F.BF.1, F.IF.3, F.IF.4, F.IF.7a, F.IF.7c, F.IF.8, F.IF.9
			CCSS ELA-Writing Standards
			W.11-12.2a and W.11-12.2b

SBAC Claims	PARCC Sub-Claims
2	D

Before the task:

- Review the concept of graphing a cubic polynomial (key features), and using a calculator for graphs and tables.
- Students will need to understand how to find the intervals of the cubic polynomial graph.
- Review the concepts of area, surface area, and volume of shapes.
- Review the concept of compound inequalities and how to write and graph them.

Vocabulary:
Compound inequalities
Constraints
Interval
Maximum/minimum point

Timeline:

There are two different options to choose.

Option 1: This option should take 3 days (or 3 hours with the assumption that math lessons/activities take up an hour during the school day).

Day 1: The students should complete Part A.
Day 2: The students should complete Part C.
Day 3: The students should complete Part D.

Option 2: This option should take 4 days (or 4 hours with the assumption that math lessons/activities take up an hour during the school day).

Day 1: The students should complete Part A.
Day 2: The students should complete Part B* (optional group activity).
Day 3: The students should complete Part C.
Day 4: The students should complete Part D.

*Part B contains group work that may take longer than expected. You may need to plan for an extra day if needed.

Other suggestions:

Part B:
Using the net provided (last page of this document) for the teacher example, show students how to construct the basic package discussed in this problem from a piece of paper and tape. To do this, cut out four congruent squares from each corner of the rectangle. Fold the sides up to form a rectangular prism. Explain how the top will be covered in clear plastic to display the product. If you have access to cardboard or a piece of poster board that will work even better.

You can also have the students make their own nets if you don't want to use the net provided.

- Have students make a package of their own using this process. Assign students different sizes of squares to be removed from each corner. A good way to break up the class would be to number each student in each group from 1 to 7, and then assign a square length:
  - All "1" students will make a package with square side lengths of 0.5 inch each.
  - All "2" students will make a package with square side lengths of 1 inch each.
  - All "3" students will make a package with square side lengths of 1.5 inches each.
  - All "4" students will make a package with square side lengths of 2 inches each.
  - All "5" students will make a package with square side lengths of 2.5 inches each.
  - All "6" students will make a package with square side lengths of 3 inches each.
  - All "7" students will make a package with square side lengths of 3.5 inches each.

You may want to assign 1-2 students a square side length of 4 inches. This may be a good example of when a rectangular prism can't be constructed because the side length is too large. It may also help when the students are deciding what the side value of the squares will be in the task. There will be two solutions found, but only one solution will work.

When finished, have a volunteer from each size square show his or her group's package to the class so that students can visualize how changing the size of the square results in packages of significantly different shapes.

Suggest that the students go back and check their work in Part A. There may have been some confusion in using dimensions and what the display portion meant. They may have a better understanding of that after this group activity, and they should be encouraged to go back and fix any errors they may have made in Part A.

Part C:
The use of a graphing calculator would be very helpful to the students.

In Question 5, encourage the students to write their answer in standard polynomial form even though it is not specifically stated.

In Questions 6 and 7, the students would be able to find the key features (including the maximum point) if they used a graphing calculator. If students aren't familiar with the functionality of the graphing and table windows, this would be a great teaching moment for them.

Extension Activity

If time permits, introduce more information about selling the product in the package. Include quantities that have been sold in the past and how many may be projected to be sold this year with any changes that were suggested by the student.

For example:
The company sold 5,000 units of the product each month with a net profit of $2.50 per unit last year. The research team informed the company that for each increase of 15 in^2 of surface area of the original display portion, sales would increase by 130 units per month, but the net profit drops by $0.17 per unit. In a similar fashion, for each decrease of 15 in^2 of surface area, the sales decrease by 130 units per month, but the net profit increases by $0.17 per unit. Use this information to find the projected profit the company will make if they use the package dimensions you suggested in the task.

Scoring Rubric

Part A

4 Point Response:

The response demonstrates a high level of understanding.

The response demonstrates:

- A strong ability to make sense of a real-world problem and develop a solution that meets given requirements;
- A strong ability to calculate accurately with an appropriate degree of precision;
- A strong ability to check work and communicate reasoning in a clear and precise way;
- A strong understanding of how to solve real-world and mathematical problems leading to writing and graphing equations with one variable.

A level 4 response should include:

- A clear and correct explanation or work that shows how to find each expression that represents the dimensions of the package;
- Correct inequalities that represent the constraints for each dimension;
- A correct compound inequality that represents the constraints on the possible values of x;
- A clear and complete explanation or work that shows how the compound inequality that represents the constraints on the possible values of x is found;
- Correct dimensions for the squares that are to be cut out of the cardboard;
- A clear and complete explanation or work that shows how the square's dimensions are found;
- The correct volume of the package along with clear and complete work or explanation.

Sample Responses for Part A

Question 1:
The algebraic expressions that represent the length, width, and height of the package are:

Length: $20 - 2x$
Width: $18 - 2x$
Height: x

These were determined by using the diagram that shows the uncut cardboard with a length of 20 inches and a width of 18 inches. Four squares with a side measurement of x inches will be cut from the cardboard so that it can be folded to create a package or open-top box. Because there are 2 squares on the length side, the length of the cardboard will be decreased by $x + x$ or $2x$; thus the length is $20 - 2x$. Because there are 2 squares on the width side, the width of the cardboard will also be decreased by $x + x$ or $2x$; thus the width is $18 - 2x$. The sides will be folded so that the square side length will be the height of the package; thus the height is x.

Question 2:
The constraints for the three dimensions will be written as inequalities. Each of the dimensions, because they are measurements, will not be negative values. For this reason, all three of the expressions used to represent the dimensions will be greater than 0.

The constraints for the length of the package will be $20 - 2x > 0$. Then solve for x to find the exact constraint for the length, which is $x < 10$. The constraints for the width of the package will be $18 - 2x > 0$. Then solve for x to find the exact constraint for the width, which is $x < 9$. The constraint for the height will be $x > 0$.

The constraints for the possible values of x are found by determining the intersection of the three constraints for the dimensions. This can be found by graphing each of the constraints on a separate number line and identifying the overlaps that occurs on all three number lines. The overlap on all three constraints will be between 0 and 9. All three number lines have shading between these two values.

Even though the graphs representing $x < 10$ and $x > 0$ include the value of 10, the other graph does not include this value. The constraints for the possible values of x should be written as a compound inequality: $0 < x < 9$.

Question 3:

The surface area for the display portion is 120 in^2. This represents the top of the package with a length of $(20 – 2x)$ inches and a width of $(18 – 2x)$ inches. This display portion is a rectangle, and the area of a rectangle is length times width. The equation $(20 – 2x)(18 – 2x) = 120$ represents this area. Solve the equation for x to determine the height of the package.

$(20 – 2x)(18 – 2x) = 120$

$360 – 40x – 36x + 4x^2 = 120$

$4x^2 – 76x + 240 = 0$

$4(x^2 – 19x + 60) = 0$

$4(x – 15)(x – 4) = 0$

$x – 15 = 0$; $x = 15$

$x – 4 = 0$; $x = 4$

The height of the package could be 15 inches or 4 inches. However, 15 inches as the height would make the length and width of the package negative values. Because this is impossible, 15 is an extraneous solution in this situation and would not be a valid measurement for the squares (or the height). Therefore, the measurements for the squares being cut out are 4 inches per side.

Question 4:

Since $x = 4$, the length and width of the package can be found using substitution. The length is $20 – 2x$ where $x = 4$, so $20 – 2(4) = 12$ inches. The width is $18 – 2x$ where $x = 4$, so $18 – 2(4) = 10$ inches. Once the package is assembled, the dimensions of the box will be length = 12 inches, width = 10 inches, and height = 4 inches. The formula for the volume of a rectangular prism is length times width times height. So, the volume of the package is $(12)(10)(4) = 480$ in^3.

3 Point Response:

The response demonstrates a strong understanding, but the work is incomplete or contains minor errors.

A level 3 response is characterized by:

- A complete explanation or work that shows how each expression that represents a dimension of the package is found, but a minor error is made;
- Three inequalities that represent the constraints for each dimension, but a minor error is made;
- A compound inequality that represents the constraints on the possible values of x, but the answer is incorrect due to a minor error made in Question 2, OR a compound inequality that represents the constraints on the possible value of x, but a minor calculation error is made;
- A complete explanation or work that shows how the compound inequality that represents the constraints on the possible values of x is found, but it is not correct due to a minor error in Question 2 or a minor calculation error in Question 3;

- The dimensions for the squares that are to be cut out of the cardboard, but a minor calculation or conceptual error is made;
- A complete explanation or work that shows how the square's dimensions are found, but it is not correct due to a minor error;
- The volume of the package with work shown, but a minor calculation error is made.

2 Point Response:

The response demonstrates a basic but incomplete understanding.

A level 2 response is characterized by:

- An explanation or work that shows the expressions that represent the dimensions of the package, but two minor errors are made or a dimension is missing;
- Inequalities that represent the constraints for the dimensions, but two minor errors are made or a constraint is missing;
- An inequality that represents the constraints on the possible values of x, but the answer is incorrect due to errors made in Question 2, OR an inequality that represents the constraints on the possible value of x, but two minor calculation or conceptual errors are made;
- An explanation or work that shows how the inequality that represents the constraints on the possible values of x is found, but it is not correct due to errors in Question 2 or the minor calculation or conceptual errors in Question 3 or information is missing or incomplete;
- The dimensions for the squares that are to be cut out of the cardboard, but two minor calculation or conceptual errors are made;
- An explanation or work that shows how the square's dimensions are found, but it is not correct due to two minor errors or information is missing or incomplete;
- The volume of the package with work shown, but two minor calculation errors are made.

1 Point Response:

The response demonstrates a minimal understanding.

A level 1 response is characterized by:

- An explanation or work that shows the expressions that represent the dimensions of the package, but more than two minor errors or a major error is made or two dimensions are missing;
- Inequalities that represent the constraints for the dimensions, but more than two minor errors or a major error is made or two constraints are missing;
- An inequality that represents the constraints on the possible values of x, but it is incorrect due to more than two minor calculation errors or a major conceptual error that is made;
- An explanation or work that shows how the inequality that represents the constraints on the possible values of x is found, but it is not correct due to more than two minor calculation errors or a major conceptual error or information is missing or incomplete;
- The dimensions for the squares that are to be cut out of the cardboard, but more than two minor calculation errors or a major conceptual error is made;
- An explanation or work that shows minimal understanding of how to find the square's dimensions are found, or information is missing or incomplete;
- The volume of the package, but more than two minor calculation errors or a major calculation or conceptual error is made, or the correct volume is given but no work is shown.

0 Point Response:

There is no response, or the response is off topic.

Note: There is no rubric for Part B, which is optional.

Part C

4 Point Response:

The response demonstrates a high level of understanding.

The response demonstrates:

- A strong ability to make sense of a real-world problem and develop a solution that meets given requirements;
- A strong ability to calculate accurately with an appropriate degree of precision;
- A strong ability to check work and communicate reasoning in a clear and precise way;
- A strong understanding of how to solve real-world and mathematical problems leading to writing and graphing equations with two variables;
- A strong understanding of how to interpret, graph and analyze a cubic function.

A level 4 response includes:

- A clear and correct explanation or work that shows how to represent the volume as a function in terms of *x*;
- A correct graph of the function; with key features labeled correctly;
- Four correct intervals listed;
- A clear and complete explanation of each interval and how each interval relates to the context;
- Correct values for the maximum volume and the square cutout along with an explanation of how these were found;
- Correct work that shows the difference between the original package volume and the maximum volume; a clear and complete explanation of the comparison between the two different packages;
- The correct amount of money each cardboard cost the company, with clear and complete work shown;
- A clear and correct explanation that specifies the dimensions of the package suggested for use so that it is the most cost efficient.

Sample Responses for Part C

Question 5:

The company wants to look at other options for the square's dimensions. Using x as the square's side measurement again, the function that represents the volume of the package, $f(x)$, with the unknown height (square dimension) can be found by setting up the equation: $f(x) = x(20 - 2x)(18 - 2x)$. To write the equation in standard polynomial form, multiply the factors.

$f(x) = x(20 - 2x)(18 - 2x)$

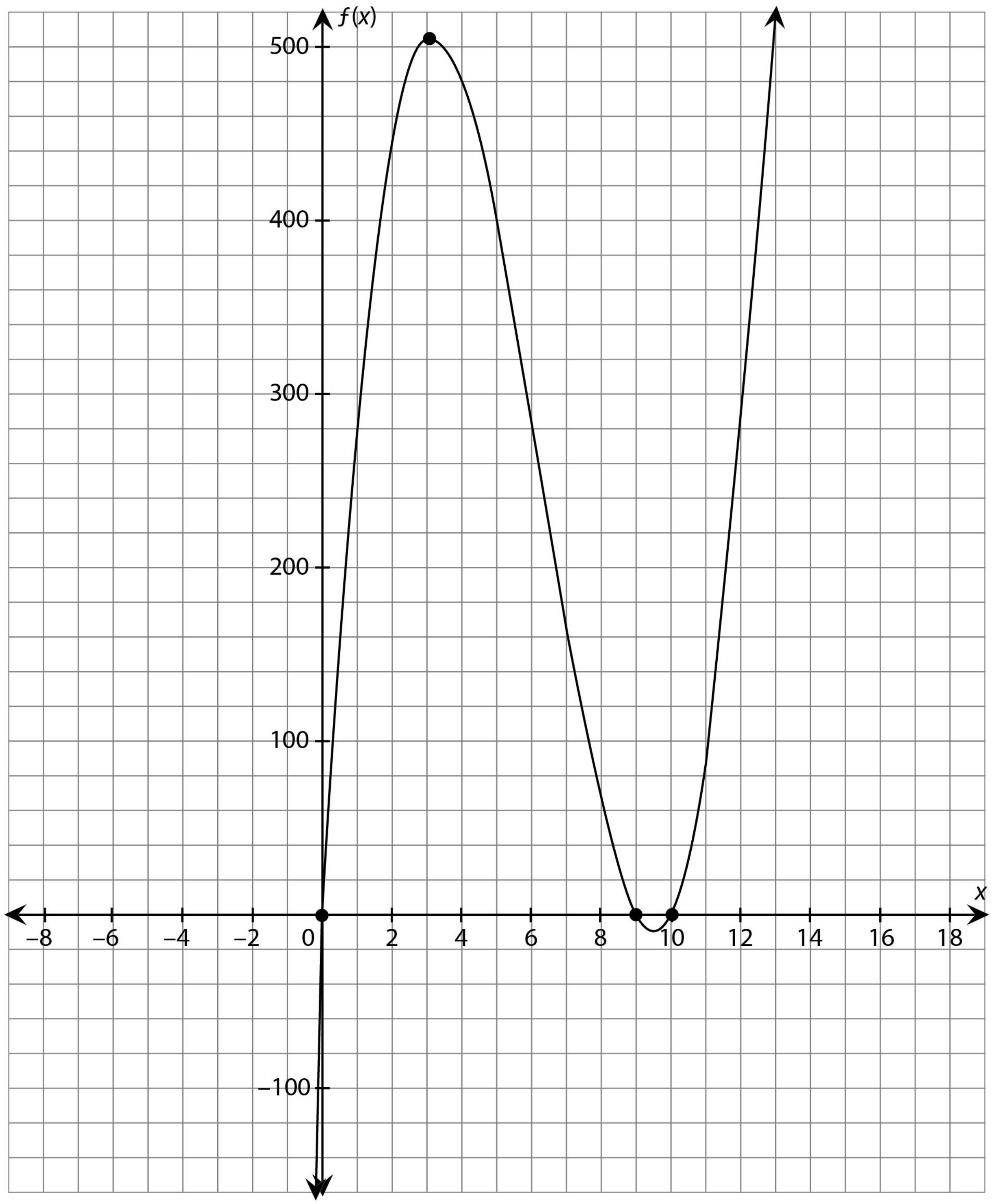
$f(x) = x(4x^2 - 76x + 360)$

$f(x) = 4x^3 - 76x^2 + 360x$

The function $f(x) = 4x^3 - 76x^2 + 360x$ represents the volume of a package with an unknown square measurement value.

Question 6:

The graph of the function $f(x) = 4x^3 - 76x^2 + 360x$ can be used to visually see the key features of the function and apply the values of these features to the context. The key features are labeled on the graph.



f(x)
500
400
300
200
100
–100
x
–8
–6
–4
–2
0
2
4
6
8
10
12
14
16
18

Question 7:

The x-intercepts are the values of x that would make the dimensions of the package equal to 0. This would then cause the package to not exist.

Each interval is defined by the section of the graph that is between the x-intercepts. Use the 3 x-intercepts and the curve of the graph, along with the end behavior, to determine how many intervals there are for this graph. There are four intervals on the graph: $x < 0$, $0 < x < 9$, $9 < x < 10$, $x > 10$.

Interval #1: $x < 0$

Looking at the curve on the graph, all of the x-values less than 0 are negative and the y-values are negative as the curve continues below the x-axis to negative infinity.

This interval is not defined for this problem because x represents the length of the square, which can't be a negative value.

Interval #2: $0 < x < 9$

The curve shows that all the values of the volume are positive since it is above the x-axis. This interval is the only part of the graph that x is defined on; therefore, it shows the volume of the package depending on the length of x.

Interval #3: $9 < x < 10$

The curve goes below the x-axis. Even though the x-values are positive, the y-values that represent the volume are negative. This interval is not defined for this problem because x represents the length of the square, and values between 9 and 10 would make a dimension negative. This results in a negative volume, which is not possible.

Interval #4: $x > 10$

The curve is above the x-axis, but because the x-values are so large, they will not work as values for x. This interval is not defined for this problem because x represents the length of the square, and values of x greater than 10 would make the dimensions of the package negative, which is not possible.

Question 8:

This curve has a maximum point within the appropriate values of x (between 0 and 9). The maximum point is approximately (3.15, 504.9). This means that when the square side measure is 3.15 inches, then the volume will be 504.9 in^3. The exact measure is found by using the graph function on the graphing calculator and either tracing the curve in the graph window or looking at the table values on the calculator, or using the max value function.

Question 9:

The maximum volume is about 505 in^3 when the square dimensions are 3.15 inches. This would make the length of the package 13.7 inches and the width 11.7 inches. The volume using the original dimensions is 480 in^3. There is a difference of about 25 in^3 in the two volumes. If each bag is 6 inches in height, 1.5 inches wide, and 4.5 inches long, then the bags would still fit in the package, but there will be a lot of air between the bags. There is not enough room to rearrange the bags so that there are more than 4 bags in a layer. Because the snacks may be fragile, it is probably a good idea to have some room between the bags. I think either using the original dimensions or the dimensions from the maximum volume would be okay.

Question 10:

The dimensions of the cardboard did not change. The company uses a piece of cardboard that is 20 inches by 18 inches. This gives an area of 360 in^2 for each piece. The company is paying \$0.005 for each square inch. The company pays \$1.80 for each piece of cardboard.

Question 11:

Using the original packaging dimensions, the square cutout has an area of 16 in^2 (4×4). There are 4 squares that are cut out, so the amount of wasted cardboard is 64 in^2 (16×4). This is costing the company \$0.32 per cardboard.

Using the packaging dimensions with the maximum volume of 505 in^3, the square cutout has an area of 9.9225 in^2 (3.15×3.15). The total amount of wasted cardboard is 39.69 in^2 (9.9225×4). This is costing the company \$0.20 per cardboard.

Another option may be to use a square cutout that has a side measurement of 1.75 inches. This would make the length of the package 16.5 inches and the width 14.5 inches. Each bag of snacks is 4.5 inches long, so 4 bags would be 18 inches. If the bags were placed in the box at a slight angle so that there was a slight overlap of the bags, then 4 bags would fit in a row instead of the 2 bags that the original package has. Then another row of 4 bags could be placed in the box the same way so that there is only one layer of bags. The height of each bag is 6 inches, so 2 bags could be placed one above the other in the one layer. The new width of the packaging is 14.5, so this gives the bags plenty of room. Since the width of the snack bag is 1.5 inches, the depth/height of the packaging has room to fit the bags evenly with the slight overlap. With this packaging, the 8 bags could all be seen in the display portion. Though this package may take up a little more room on the shelf in the stores, it won't be as tall as the original so more packages can stack on top. This would also save the company some in wasted cardboard since the square cutouts are smaller. The square cutout has an area of 3.0625 in^2 (1.75×1.75). The total amount of wasted cardboard is 12.25 in^2. This would cost the company $0.06 per cardboard.

3 Point Response:

The response demonstrates a strong understanding, but the work is incomplete or contains minor errors.

A level 3 response is characterized by:

- A complete explanation or work that shows how to represent the volume as a function in terms of *x*, but a minor calculation or conceptual error is made;
- A graph of the function, but a minor error is made; key features are labeled correctly, but one may be incorrect or missing;
- Four intervals listed, but a minor conceptual error is made;
- A complete explanation of each interval and how each interval relates to the context, but a minor conceptual error is made;
- The values for the maximum volume and the square cutout along with an explanation of how these were found, but a minor calculation error is made;
- Work that shows the difference between the original package volume and the maximum volume, but a minor calculation or conceptual error is made; a complete explanation of the comparison between the two different packages;
- The amount of money each cardboard cost the company with work shown, but a minor calculation error is made;
- A complete explanation that specifies the dimensions of the package suggested for use so that it is the most cost efficient, but a minor calculation or conceptual error is made in the explanation.

2 Point Response:

The response demonstrates a basic but incomplete understanding.

A level 2 response is characterized by:

- An explanation or work that shows how to represent the volume as a function in terms of *x*, but two minor calculation or conceptual errors are made;
- A graph of the function, but two minor errors are made; key features are labeled correctly, but 1–2 may be incorrect or missing;
- Three of the four intervals listed;
- An explanation of each interval and how each interval relates to the context, but two minor conceptual errors are made;
- The values for the maximum volume and the square cutout along with an explanation of how these were found, but two minor calculation errors are made and the explanation may be incomplete;

- Work that shows the difference between the original package volume and the maximum volume, but two minor calculation or conceptual errors are made; an explanation of the comparison between the two different packages that may have incomplete or missing information;
- The amount of money each cardboard cost the company with work shown, but two minor calculation errors are made;
- An explanation that specifies the dimensions of the package that is suggested for company use because it is the most cost efficient, but two minor calculation or conceptual errors are made in the explanation or the explanation contains incomplete or missing information.

1 Point Response:

The response demonstrates minimal understanding.

A level 1 response is characterized by:

- An explanation or work that shows how to represent the volume as function in terms of x, but more than two minor calculation or conceptual errors are made or one major error is made;
- A graph of the function, but more than two minor errors are made or one major error is made; key features are labeled correctly, but 3 or more may be incorrect or missing;
- Two of the four intervals listed;
- An explanation of each interval and how each interval relates to the context, but more than two minor conceptual errors are made or one major error is made;
- The values for the maximum volume and the square cutout along with an explanation of how these were found, but more than two minor calculation errors are made or one major error is made, and the explanation may be incomplete or missing;
- Work that shows the difference between the original package volume and the maximum volume, but more than two minor calculation or conceptual errors are made or one major error is made; an explanation of the comparison between the two different packages that may have incomplete or missing information;
- The amount of money each cardboard cost the company with work shown, but more than two minor calculation errors are made or one major error is made;
- An explanation that specifies the dimensions of the package that is suggested for company use because it is the most cost efficient, but more than two minor calculation or conceptual errors are made, or one major error is made in the explanation, or the explanation contains incomplete or missing information.

0 Point response:

There is no response, or the response is off topic.

Part D

4 Point Response:

The response demonstrates a high level of understanding.

The response demonstrates:

- A strong ability to make sense of a real-world problem and develop a solution that meets given requirements;
- A strong ability to check work and communicate reasoning in a clear and precise way;
- A strong ability to justify the solution and communicate this to others;
- A strong ability to calculate accurately with an appropriate degree of precision;
- A strong understanding of how to solve real-world and mathematical problems leading to writing and graphing equations with two variables;
- A strong understanding of how to compare two or more different functions that are shown verbally or graphically and state their differences.

A level 4 response should include:

- A detailed report that clearly explains which packaging is the most ideal for the company's product and store display and is also cost effective. All of the information that was used in the task should be included in this report, such as: the measurements of the current packaging; how the current packaging is working for the company; examples of measurements that could be used for new packaging dimensions; explanations of how the measurements were calculated; how the product fits into the current package compared to other packaging options; cost efficiency pertaining to waste after the cardboard is cut; fact-based reasons for recommended packaging designs; all supporting documents that provide the background information and work, including calculations;
- A report that contains the reasoning behind the choices made by the student; the choices are strongly supported with at least 8 sentences that clearly demonstrate a strong understanding of the thought process involved in making these decisions.

3 Point Response:

The response demonstrates a strong understanding, but the work is incomplete or contains minor errors.

A level 3 response is characterized by:

- A report that explains which packaging is the most ideal for the company's product and store display and is also cost effective. All of the information that was used in the task should be included in this report, such as: the measurements of the current packaging; how the current packaging is working for the company; examples of measurements that could be used for new packaging dimensions; explanations of how the measurements were calculated; how the product fits into the current package compared to other packaging options; cost efficiency pertaining to waste after the cardboard is cut; fact-based reasons for recommended packaging designs; all supporting documents that provide the background information and work, including calculations; the report may contain 1–2 minor errors or may have 1–2 incomplete specifics;
- A report that contains the reasoning behind the choices made by the student; the choices are supported with at least 7 sentences that demonstrate a strong understanding of the thought process involved in making these decisions, but 1–2 ideas are incomplete or incorrect due to minor errors made in the calculations.

2 Point Response:

The response demonstrates a basic but incomplete understanding.

A level 2 response is characterized by:

- A report that explains which packaging is the most ideal for the company's product and store display and is also cost effective. All of the information that was used in the task should be

included in this report; all supporting documents that provide the background information and work, including calculations; the report may contain more than two minor errors or one major error or may have three incomplete specifics or 1–2 missing specifics;
- A report that contains the reasoning behind the choices made by the student; the choices are supported with at least 5–6 sentences that demonstrate a basic understanding of the thought process involved in making these decisions with 2–3 ideas being incomplete or incorrect due to minor errors made in the calculations.

1 Point Response:

The response demonstrates minimal understanding.

A level 1 response is characterized by:
- A report that explains which packaging is the most ideal for the company's product and store display and is also cost effective. Information that was used in the task should be included in this report; all supporting documents that provide the background information and work, including calculations; the report may contain two major errors or may have four or more incomplete specifics or 3–4 missing specifics;
- A report that contains the reasoning behind the choices made by the student; the choices are supported with 4 sentences that demonstrate a minimal understanding of the thought process involved in making these decisions with 3 or more ideas being incomplete or incorrect due to minor errors made in the calculations.

0 Point response:

There is no response, or the response is off topic.