Inspect
CCR Performance Tasks

Algebra II: Make Sense of Problems Using Exponential and Logarithmic Functions

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Inspect offers the following assessment products:

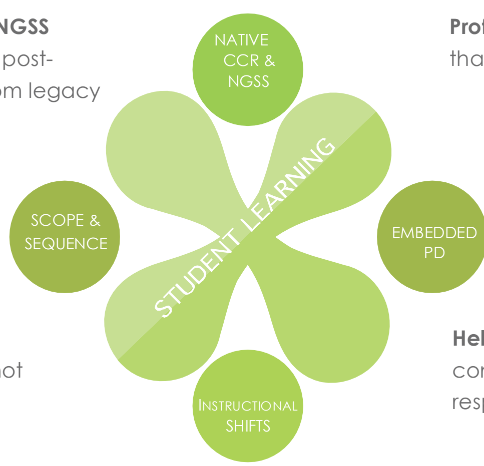
Content Bank for English/Language Arts and Math Grades 2 – High School	▪ More than 36,000 items ▪ More 1500 complex texts, including authentic permissioned texts ▪ Includes Literacy in History, Social Science, Science, and Technical Subjects
Quick Checks for English/Language Arts and Math Grades 2 – High School	▪ Fixed-form assessments with five to seven items including constructed response ▪ Key instructional concepts embedded in standards (clusters for Math, staircase of text complexity for ELA)
Focused Interim Assessments for English/Language Arts and Math Grades 3 – High School	▪ Prebuilt assessments with up to 15 items that focus on groups of related standards within a Claim or domain ▪ More focused than summative assessments ▪ Flexible and customizable ▪ Mirrors SBAC IAB blueprints
NGSS Formative Assessments Grades 5 – High School	▪ Prebuilt assessments with items linked to experimental contexts that assess the three dimensions of science learning ▪ Flexible and customizable ▪ Addresses the California Course Models and NGSS Bundles
Observational Tasks for English/Language Arts and Math Grades K - 1	▪ Developmentally appropriate for individual students and small groups

Inspect Assessment Content is available through a variety of assessment administration and data analysis platforms.

Inspect assessment content offers these benefits:

Native college- and career-ready and NGSS content prepares students to meet their post-secondary goals. Content re-aligned from legacy standards cannot do this.

Content that addresses your scope and sequence so that your assessments do not waste valuable instruction time



Professional development embedded within content that

- shows the relationship between specific skills and higher-order thinking
- includes authentic, permissioned texts of appropriate complexity
- and documents student progress using DOK and learning progressions

Help for teachers addressing the instructional shifts with content that elicits evidence of learning from each response

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CCR Performance Tasks

Algebra II: Make Sense of Problems Using Exponential and Logarithmic Functions

Student Test Booklet

Name:

Student Rubric

This problem is meant to test if you can:
- Make sense of the problem and develop a strategy to solve the problem;
- Use mathematical tools such as functions and graphs to solve the problem;
- Reason about mathematical answers in the context of the problem.

Your teacher will rate your answer as a level 4, 3, 2, 1, or 0. The descriptions below explain the types of answers expected at each level.

Level 4:
Your answer is correct and complete.
Your answer includes:
- A correct answer and correct and complete work or correct and complete explanation so your teacher understands how you solved the problem in part A;
- A correct answer and correct and complete work or correct and complete explanation so your teacher understands how you solved the problem in part B;
- A correct and complete explanation of what your answer in part B means in terms of the context of the problem.

Level 3:
Your strategy to solve the problem is correct but you have made some minor errors in your calculations or your explanation is incomplete.
Your answer includes:
- Work or explanation that shows that you understand the problem and how to solve it, and an answer consistent with your calculations which may contain minor errors. This could describe your answer in part A, part B, or both;
- An explanation of your answer in part B that shows that you understand the meaning of the answer in the context of the problem.

Level 2:
You have shown some basic understanding of the problem but you are not able to solve it completely, or your strategy is not clear to the teacher or contains errors.
Your answer includes:
- Work or an explanation that shows that you understand one part of the problem and how to solve it, and an answer consistent with your calculations which may contain minor errors; OR
- Correct answers in both parts A and B but not enough work or an insufficient explanation to make the strategy you used clear to your teacher;
- A missing or weak explanation of the meaning of your answer in the context of the problem.

Level 1:
You have shown some work that is relevant to the problem, but your answers are incorrect and you do not know how to solve the problem.

Level 0:
Your answer is not related to the question, the teacher cannot understand your answer, or you did not write anything.

Name:______________________________

Algebra II: Make Sense of Problems Using Exponential and Logarithmic Functions

Complete all the tasks in the test booklet.

1 **The population of a town is decreasing by 3% per year. The population was 5,000 in 2011.**

A. Assuming the decreasing trend continues, what will the approximate population be in the year 2021? Show your work or explain your answer.

Go On

B. Assuming the decreasing trend continues, in approximately how many years will the town have no population? Show your work and explain your answer.

STOP

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Algebra II: Make Sense of Problems Using Exponential and Logarithmic Functions

Teacher Guide

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About the Teacher Guide

This document contains support materials for "Algebra II: Make Sense of Problems Using Exponential and Logarithmic Functions."

This includes:

(a) The task
(b) The standards and depth of knowledge level of the task
(c) The scoring rubric
(d) Discussion questions
(e) Extension activities

These specifications have been included to help you connect the task to the Common Core content standards and the standards for mathematical practice. The rubric is designed to help you look for the development of mathematical practices in student work. It is also here to help you look for consistencies in student content errors that can help guide intervention and re-teach strategies.

Test Definition File

Item #	Correct Answer	Practice Standard	Content Standards
1	See Scoring Rubric	Mathematical Practice 1	F-IF.4, F-LE.1, F-LE.2, F-LE.4

SBAC Claims	PARCC Sub-Claims
1 and 2	A and D

Performance Task

The population of a town is decreasing by 3% per year. The population was 5,000 in 2011.

A. Assuming the decreasing trend continues, what will the approximate population be in the year 2021? Show your work or explain your answer.

B. Assuming the decreasing trend continues, in approximately how many years will the town have no population? Show your work and explain your answer.

Standards Alignment

Practice Standards

MP1 > DOK 3

Make sense of problems and persevere in solving them. -- Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

Content Standards

F-IF.4

For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*

F-LE.1

Distinguish between situations that can be modeled with linear functions and with exponential functions.

F-LE.2

Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

F-LE.4

For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

SBAC Claims

Mathematics Claim #1:
Concepts and Procedures. Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency.

Mathematics Claim #2:
Problem Solving. Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem solving strategies.

PARCC Sub-Claims

Sub-Claim A:
Major Content with Connections to Practices. The student solves problems involving the Major Content for her grade/course with connections to the Standards for Mathematical Practice.

Sub Claim D:
Highlighted Practice MP.4 with Connections to Content: modeling/application. The student solves real-world problems with a degree of difficulty appropriate to the grade/course by applying knowledge and skills articulated in the standards for the current grade/course (or, for more complex problems, knowledge and skills articulated in the standards for previous grades/courses), engaging particularly in the Modeling practice, and where helpful making sense of problems and persevering to solve them (MP.1), reasoning abstractly and quantitatively (MP.2), using appropriate tools strategically (MP.5), looking for and making use of structure (MP.7), and/or looking for and expressing regularity in repeated reasoning (MP.8).

Scoring Rubric

4 Point Response:
The student response demonstrates:

- A strong ability to make sense of problems and develop an appropriate solution strategy, and the ability to implement the strategy correctly;
- A strong understanding of exponential and logarithmic functions;
- A strong ability to reason about mathematical answers in the context of a problem.

A level 4 response should include:

- The correct answer in part A (3,687);
- Correct and complete work or explanation for part A;
- A correct answer in part B (about 280 years);
- Correct and complete work for part B;
- A correct and complete explanation of solution strategy and meaning of answer in part B.

A sample level 4 response looks like the following.

Part A: "$P = 5{,}000(0.97)^n = 5{,}000(0.97)^{10} = 5{,}000(0.7374) = 3{,}687$. The population will be about 3,687 people in 2021."

Part B: "The exponential function never reaches zero. But a population of less than 1 means that the town really has no population, since you can't have part of a person. So I figured out when the function predicts a population of less than one. $5000(0.97)^n < 1$, so $(0.97)^n < 1/5000$ and $n\log(0.97) < \log(0.0002)$, so $n > 279.63$. This means that the town will be empty in about 280 years as long as the population continues to decrease at 3% per year. But in reality, it is likely to speed up at the end since not many people want to be the only person in a town."

3 Point Response:
The student response demonstrates:

- A strong ability to make sense of the problem and develop a solution strategy, but there may be minor errors in the implementation of the strategy;
- A strong understanding of exponential and logarithmic functions;
- A strong ability to reason about mathematical answers in the context of a problem, but the explanation in part B may be incomplete.

2 Point Response:
The student response demonstrates:

- A basic ability to make sense of problems and develop an appropriate solution strategy;
- A basic understanding of exponential and logarithmic functions;
- A basic ability to reason about mathematical answers in the context of a problem.

A level 2 response likely includes a correct response to only part A. In part B, the student may get stuck on making sense of the question. For example, the student may not know how to interpret the fact that the exponential function never reaches 0. Another possibility is that the student figures out how to approach the problem but gets stuck trying to implement the strategy due to a weakness in technical skills. The student may not understand the relationship between logarithms and exponents and thus may not be able to solve the inequality.

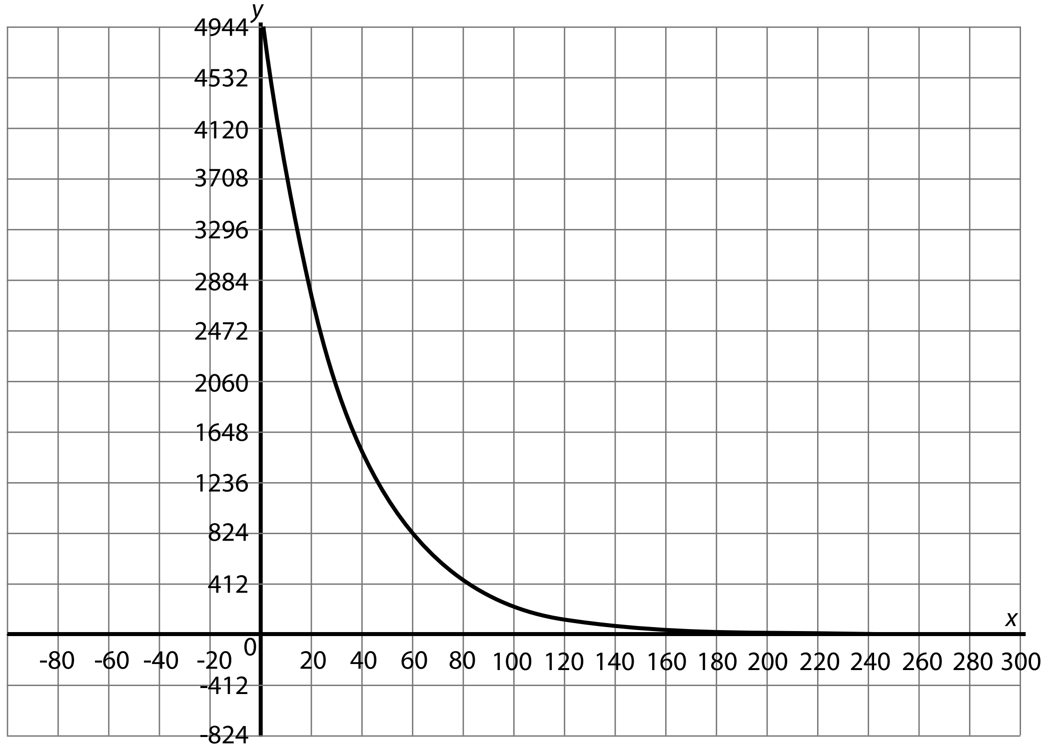
1 Point Response: The student response demonstrates: • A weak ability to make sense of the problem and develop an appropriate solution strategy. The student may make an attempt at part A and get stuck or make errors. Part B may be missing or incorrect; • A weak understanding of exponential and logarithmic functions; • A weak ability to reason about mathematical answers in the context of a problem.
0 Point Response: Student provides no response, or response is off topic.

Discussion Questions

Use the following questions to stimulate discussion:

1. How does this situation translate into graphic form?

Possible Response: *The function to graph is* $y = 5000(0.97)^x$ *where y represents the population x years after 2011.*



2. What would a negative *x*-value represent?

Possible response: *A negative value of x represents the number of years before 2011; for example, x=-5 represents 2006, and the corresponding y-value (5,822) represents the population in that year, according to the model.*

3. How realistic is this scenario?

Possible response: *Most population rates of increase or decrease are estimates or averages over time. It is unlikely that the model fits reality accurately at any particular point in time.*

4. Explain why the general model for population decrease is very likely to deviate from reality as time goes by.

Possible response: *Once the population of a town falls below a certain level, the rate of decrease will not slow down, as shown in the exponential model, but would likely speed up, as the few people left in the town would not remain in a town that is essentially empty. Also, this model ignores all outside factors that can influence population change, such as the creation of new jobs in the area.*

Extension Activities

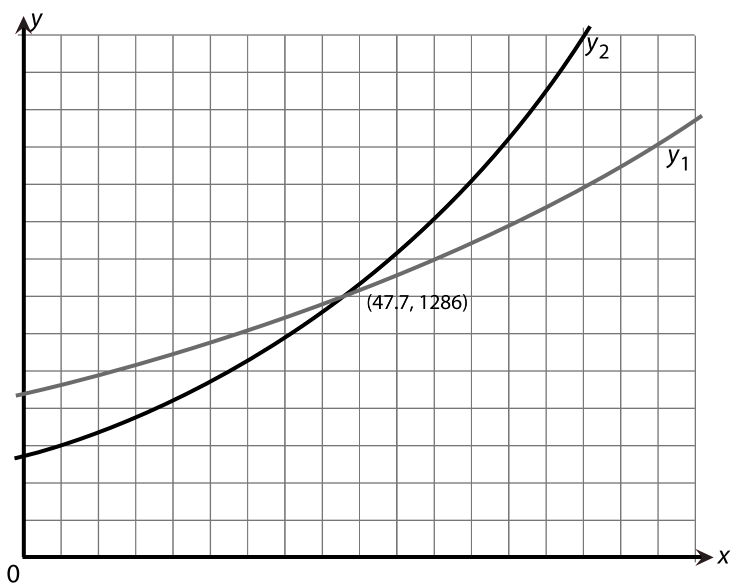
1. Determining Appropriate Graphing Windows.

This extension activity can be used with graphing calculators or with graph paper. If graphing technology is not available it will be helpful to at least have some sample images available to show students.

Abigail invests $800 in an account that earns 1% annual interest. Brady invests $500 in an account that earns 2% annual interest. If each person leaves the account alone, making no further deposits or withdrawals, how many years will it take for Brady's account to have more money than Abigail's account?

A. Set up the equations representing the situation.
Abigail: $y_1 = 800(1.01)^x$
Brady: $y_2 = 500(1.02)^x$ where y is the amount in the account after x years.

B. Determine a window that shows the most appropriate section of the graph.
Note that there are various methods to achieve this goal, including solving the system to determine the intersection point, creating a table of values, etc. Once the intersection point is established (47.7, 1286) then a reasonable amount of the graph should be shown on either side of the intersection. Key features of the graph that should be shown are (1) the intersection, (2) the y-intercepts, and (3) enough graphical space past the intersection point to make it clear that $y_2 > y_1$ for $x > 47.7$.



x min: -10
x max:100
y min: 0
y max: 2600

C. Discuss what happens to the graph when other windows are used that include the intersection point, such as:
$40 \le x \le 50;\ 0 \le y \le 5000$
$0 \le x \le 100;\ 1280 \le y \le 1290$
$0 \le x \le 500;\ 0 \le y \le 500$
$0 \le x \le 1500;\ 0 \le y \le 1500$

2. Using Successive Approximation with Tables.

> The half-life of nickel-63 is 96 years. A scientist has 15 grams of nickel-63. How long will it take for there to be less than 3 grams of nickel remaining?

A. Determine the function to model the situation.

$y=15\left(\frac{1}{2}\right)^{\frac{x}{96}}$ where y is the amount of nickel-63 remaining after x years.

B. Estimate the solution.

$15 \text{ x } \frac{1}{2} \text{ x } \frac{1}{2} = 3.75$ and $15 \text{ x } \frac{1}{2} \text{ x } \frac{1}{2} \text{ x } \frac{1}{2} = 1.875$ so it will take somewhere between 2 and 3 half-life time periods. Each time period is 96 years, so the solution is between 96(2) = 192 years and 96(3) = 288 years.

C. Create a table of values to span the range needed to find where y is first less than 3.

x	*y*
200	3.54
220	3.06
240	2.65
260	2.29

x must fall between 220 and 240

D. Create additional tables, shrinking the change in *x* each time.

x increases by 5:

x	*y*
220	3.06
225	2.96
230	2.85

x must fall between 220 and 225

x increases by 1:

x	*y*
220	3.06
221	3.04
222	3.02
223	2.998
224	2.98

x must fall between 222 and 223

Note: Discuss why it's necessary to list the value of y to 3 decimal places here (rounding up correctly would show that the value is 3, when it is really below 3).

x increases by 0.1:

x	*y*
222.1	3.0197
222.2	3.0175
222.3	3.0153
222.4	3.0131
222.5	3.0110
222.6	3.0088
222.7	3.0066
222.8	3.0044
222.9	3.0001
223.0	2.9979

x must fall between 222.9 and 223.0

E. Discuss the reasons for obtaining progressively more precise approximations or for stopping at particular levels of precision.

F. Compare the “final” approximation with the algebraic solution to the problem, pointing out how using the successive approximation method to tenths creates a solution that is accurate to the ones’ place.

The final approximation above gives $y<2.9979$ when $x>223.0$, while the algebraic solution is:

$$15\left(\frac{1}{2}\right)^{\frac{x}{96}} < 3$$

$$\left(\frac{1}{2}\right)^{\frac{x}{96}} < \left(\frac{3}{15}\right)$$

$$\log\left(\frac{1}{2}\right)^{\frac{x}{96}} < \log\left(\frac{3}{15}\right)$$

$$\left(\frac{x}{96}\right)\log\left(\frac{1}{2}\right) < \log\left(\frac{3}{15}\right)$$

$$x > 96\left[\frac{\log\left(\frac{3}{15}\right)}{\log\left(\frac{1}{2}\right)}\right]$$

$$x > 222.90509$$