Inspect
CCR Performance Tasks

Algebra II: Model Sound Waves

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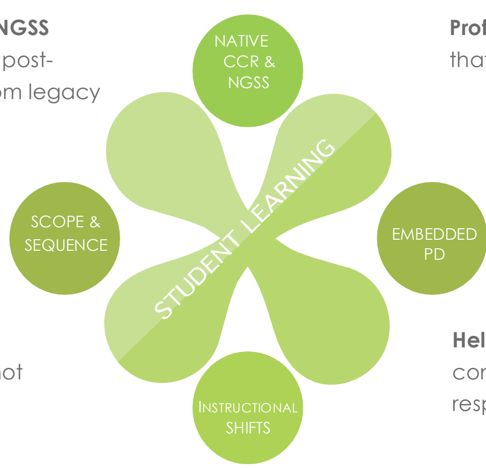

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- shows the relationship between specific skills and higher-order thinking
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- and documents student progress using DOK and learning progressions

Content that addresses your scope and sequence so that your assessments do not waste valuable instruction time

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CCR Performance Tasks

Algebra II: Model Sound Waves

Student Test Booklet

Name:

Student Rubric

This problem is meant to test if you can:
• Interpret periodic functions as models of sound waves;
• Find the frequency and period of sine functions;
• Write equations and sketch graphs of sine functions with given frequencies.

Your teacher will rate your answer as a level 4, 3, 2, 1, or 0. The descriptions below explain the types of answers expected at each level.

Level 4:
Your answer is correct and complete. Your answer includes:
• Correct determinations of period and frequency for specific sine functions with work or an explanation;
• A correct equation for a sine function with a given frequency with work or an explanation;
• Correct sketches of sine functions modeling sounds with different pitches and loudnesses;
• Correct equations and sketches of graphs of sine functions with given frequencies;
• Correct identification and interpretation of the sections of a graph that represent constructive and destructive interference;
• An accurate interpretation of a graph of a function in terms of the loudness of the sound it represents;
• Correct identification of the period of a complicated periodic function;
• Correct identification of the time between acoustic beats with a correct and complete explanation.

Level 3:
Your equations, interpretations, and sketches are mostly correct but you have made some minor errors or your explanations and work are incomplete. Your answer includes:
• Correct determinations of period and frequency for specific sine functions, but your work may be incomplete or missing;
• A correct equation for a sine function with a given frequency, but your work may be incomplete or missing;
• Correct sketches of sine functions modeling sounds with different pitches and loudnesses;
• Correct equations and generally correct but possibly inaccurate graphs of sine functions of given frequencies;
• Correct identification of the sections of a graph that represent constructive and destructive interference, but your interpretation may be incomplete or missing;
• A generally correct interpretation of a graph of a function in terms of the loudness of the sound it represents;
• An incorrect identification of the period of a complicated periodic function;
• An incorrect determination of the time between acoustic beats.

Level 2:
You have shown some basic understanding of how to use sine functions but your analysis is incomplete or contains several errors. Your answer may include:
• Correct determinations of period and frequency for specific sine functions, but your work may be incomplete or missing;
• A correct equation for a sine function with a given frequency, but your work may be incomplete or missing;
• Partially correct sketches of sine functions modeling sounds with different pitches and loudnesses;
• Partially correct equations and graphs of sine functions of given frequencies;
• Incorrect or partially correct identification of the sections of a graph that represent constructive and destructive interference, but your interpretation is incomplete, incorrect, or missing;
• An incorrect or missing interpretation of a graph of a function in terms of the loudness of the sound it represents;
• An incorrect identification of the period of a complicated periodic function;
• An incorrect determination of the time between acoustic beats.

Level 1:
You have shown little understanding of how to use sine functions. Your answer may include:
- Incorrect or missing determinations of period and frequency for specific sine functions;
- An incorrect equation for a sine function with a given frequency;
- Incorrect sketches of sine functions modeling sounds with different pitches and loudnesses;
- Incorrect equations and sketches of graphs of sine functions with given frequencies;
- Incorrect identification and interpretation of the sections of a graph that represent constructive and destructive interference;
- An incorrect interpretation of a graph of a function in terms of the loudness of the sound it represents;
- Incorrect identification of the period of a complicated periodic function;
- An incorrect determination of the time between acoustic beats.

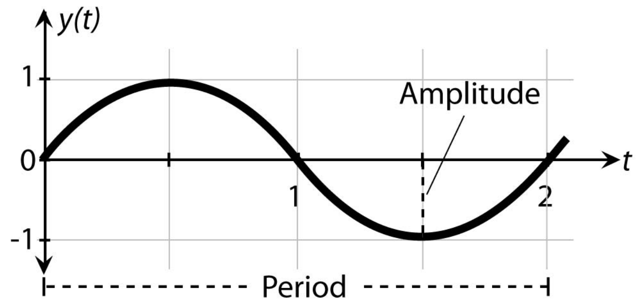
Level 0:
Your answer is not related to the question, the teacher cannot understand your answer, or you do not write anything.

Name:______________________________

Algebra II: Model Sound Waves

Complete all the tasks in the test booklet.

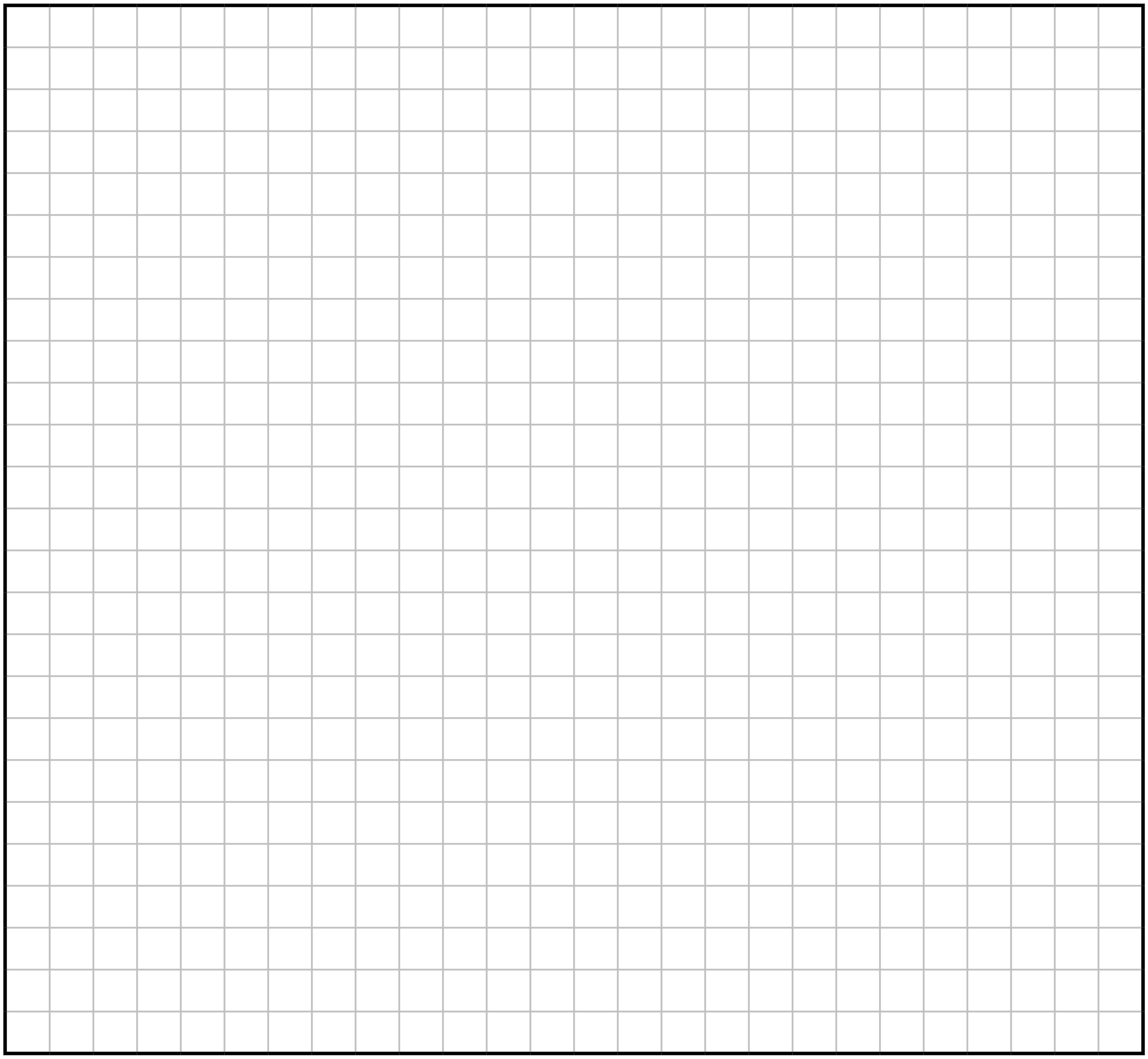
1 **Sound is a type of wave that travels through a medium such as air. Molecules of the medium vibrate back and forth to transfer wave energy through the medium. Mathematically, sound waves can be modeled as functions of time (t) by periodic functions, such as sine and cosine. Sound waves are usually described by their frequencies. Some important aspects of a sound wave are the frequency, period, and amplitude. The figure below shows the function $y(t) = \sin(\pi t)$. The period is 2 seconds, the frequency is 0.5 Hz (Hz = $\frac{1}{s}$), and the amplitude is 1 unit.**



A. Each of the following functions represents a single sound wave. For each function, determine the period in seconds and the frequency in Hertz. Show your work or explain your reasoning.

1. $y(t) = \sin(4\pi t)$
2. $y(t) = \sin(2\pi t)$

Go On



Go On

B. The musical note that is used for tuning many orchestral instruments has a frequency of 440 Hz. Write a sine function that represents a sound with a frequency of 440 Hz. Show your work or explain your reasoning.

Go On

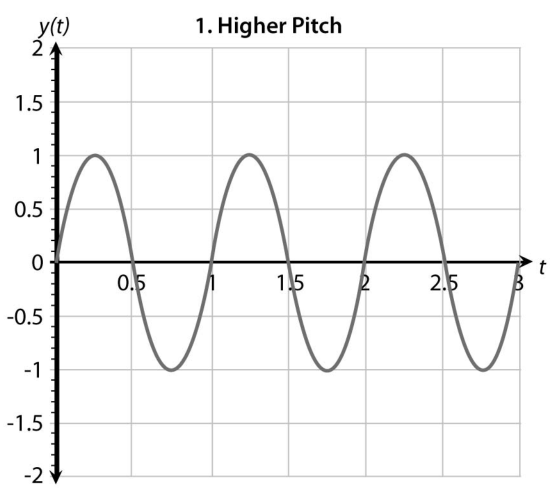
Name:______________________________

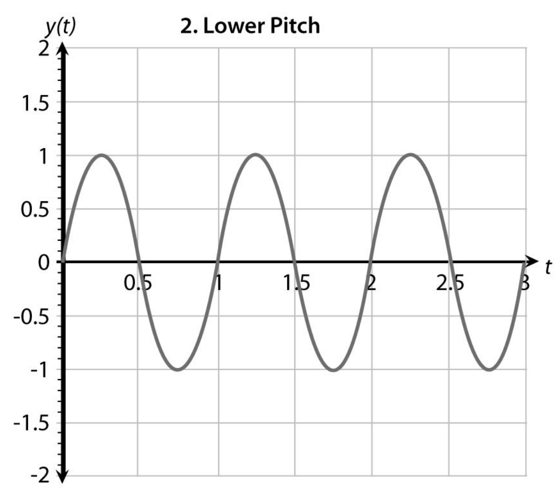
Pitch and Loudness

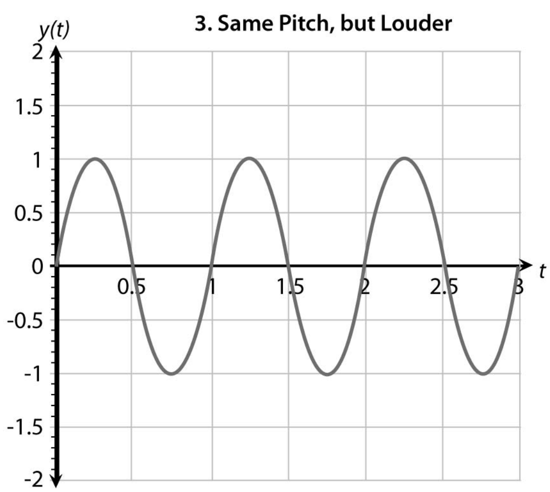
Using a sine function to model a sound allows us to represent the pitch of the sound and the loudness of the sound. Sounds with high pitches, such as bird calls, have high frequencies, while sounds with low pitches, such as fog horns, have low frequencies. The loudness of the sound has to do with how much force the molecules exert as they vibrate, and is represented by the amplitude of the sine function. The louder the sound, the greater the amplitude.

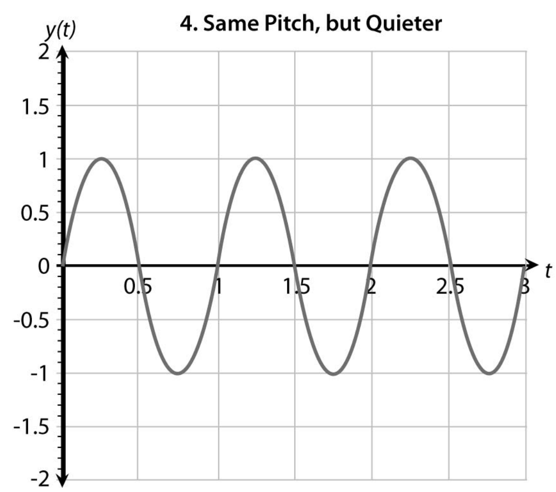
C. The grids below show a sine function that represents a particular sound. On the same grids, sketch a sine function that represents a sound with each of the following:

1. **a higher pitch**
2. **a lower pitch**
3. **the same pitch but louder**
4. **the same pitch but quieter**









Go On

Acoustic Beats

When two musical notes that are very close in pitch are played at the same time, they sound like one note (instead of being perceived as two separate notes), with a periodic variation in loudness. The loudness of the sound varies quickly between loud and so quiet that no sound is heard, at regular intervals of time. This phenomenon is known as *acoustic beats* and can be modeled by adding together the sine functions that represent each note.

D. Continue this investigation of beats.

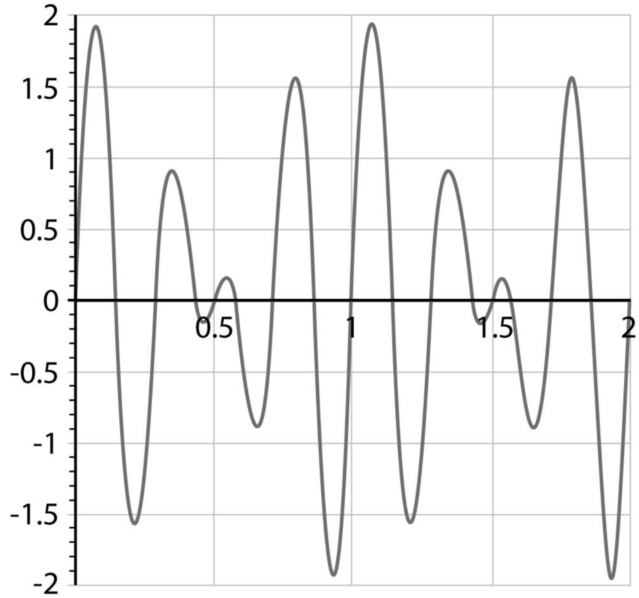
1. **Write sine functions that represent two notes that have the same loudness: one with a frequency of 3 Hz and one with a frequency of 4 Hz.**
2. **Sketch the graphs of these two functions on the grid below. Use a different color for each function. (Hint: Set the scale for the *t*-axis equal to 0-2 seconds.)**
3. **When both functions have positive values, the result is *constructive interference* of the sound waves. When one function is positive and the other is negative, the result is *destructive interference*. Mark the parts of the graph where there is constructive and destructive interference between the two sound waves.**
4. **Describe what happens to the sound at points of constructive and destructive interference.**

Go On

Name:______________________________

Algebra II: Model Sound Waves

E. The graph below shows the result of adding the sine functions for 3 and 4 Hz.



- **At how many seconds do you expect the sound to be most quiet?**
- **At how many seconds do you expect the sound to be loudest?**
- **What is the period of this combined function?**
- **How many seconds do you expect between acoustic beats? Explain your answer.**

STOP

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CCR Performance Tasks

Algebra II: Model Sound Waves

Teacher Guide

About the Teacher Guide

This document contains support materials for “Algebra II: Model Sound Waves.”
This includes:

(a) The task
(b) The standards and depth of knowledge level of the task
(c) The scoring rubric
(d) Discussion questions
(e) Extension activities

These specifications have been included to help you connect the task to the Common Core content standards and the standards for mathematical practice. The rubric is designed to help you look for the development of mathematical practices in student work. It is also here to help you look for consistencies in student content errors that can help guide intervention and reteach strategies.

Test Definition File

Item #	Correct Answer	Practice Standard	Content Standards
1	See Scoring Rubric	Mathematical Practice 4	F-TF.5

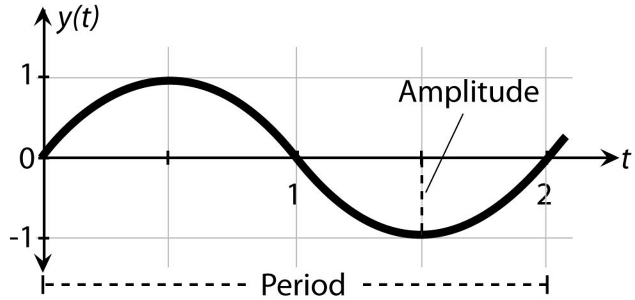
SBAC Claims	PARCC Sub-Claims
1 and 4	A and D

Special Instructions

Students need two colors of pens or pencils to complete part D.

Performance Task

Sound is a type of wave that travels through a medium such as air. Molecules of the medium vibrate back and forth to transfer wave energy through the medium. Mathematically, sound waves can be modeled as functions of time (t) by periodic functions, such as sine and cosine. Sound waves are usually described by their frequencies. Some important aspects of a sound wave are the frequency, period, and amplitude. The figure below shows the function $y(t) = \sin(\pi t)$. The period is 2 seconds, the frequency is 0.5 Hz (Hz = $\frac{1}{s}$), and the amplitude is 1 unit.



A. Each of the following functions represents a single sound wave. For each function, determine the period in seconds and the frequency in Hertz. Show your work or explain your reasoning.

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2. $y(t) = \sin(2\pi t)$

B. The musical note that is used for tuning many orchestral instruments has a frequency of 440 Hz. Write a sine function that represents a sound with a frequency of 440 Hz. Show your work or explain your reasoning.

Pitch and Loudness

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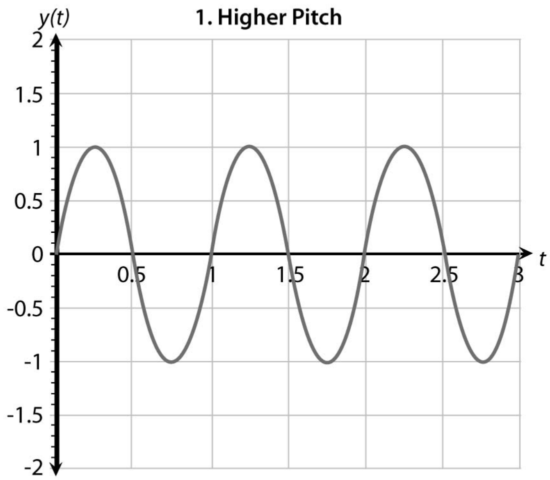
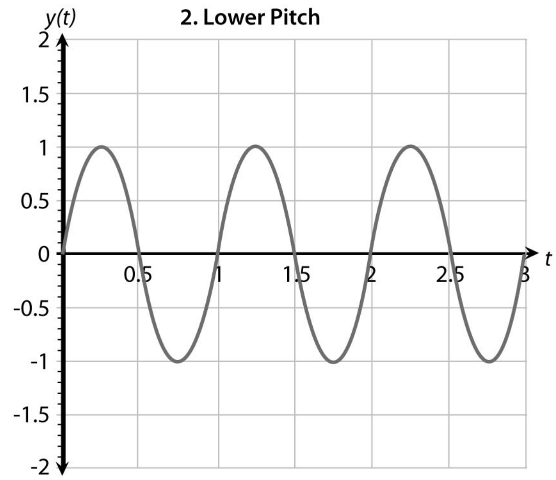
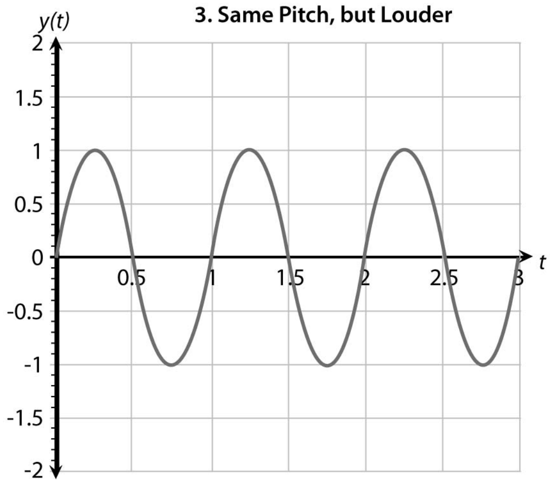
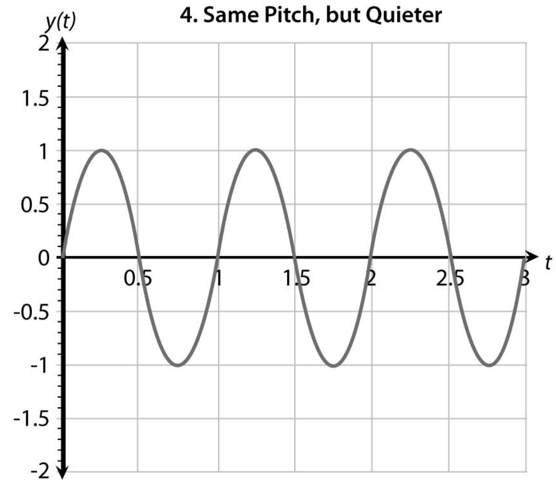
C. The grids below show a sine function that represents a particular sound. On the same grids, sketch a sine function that represents a sound with each of the following:

1. **a higher pitch**
2. **a lower pitch**
3. **the same pitch but louder**
4. **the same pitch but quieter**


1. Higher Pitch
y(t)
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0.5
1
1.5
2
2.5
3
t


2. Lower Pitch
y(t)
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0.5
1
1.5
2
2.5
3
t


3. Same Pitch, but Louder
y(t)
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0.5
1
1.5
2
2.5
3
t


4. Same Pitch, but Quieter
y(t)
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0.5
1
1.5
2
2.5
3
t

Acoustic Beats

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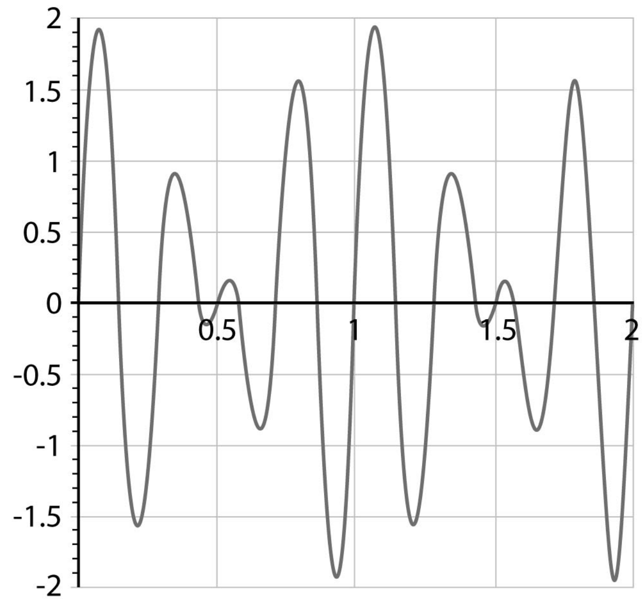
D. Continue this investigation of beats.

1. **Write sine functions that represent two notes that have the same loudness: one with a frequency of 3 Hz and one with a frequency of 4 Hz.**
2. **Sketch the graphs of these two functions on the grid below. Use a different color for each function. (Hint: Set the scale for the *t*-axis equal to 0-2 seconds.)**
3. **When both functions have positive values, the result is *constructive interference* of the sound waves. When one function is positive and the other is negative, the result is *destructive interference*. Mark the parts of the graph where there is constructive and destructive interference between the two sound waves.**
4. **Describe what happens to the sound at points of constructive and destructive interference.**

E. The graph below shows the result of adding the sine functions for 3 and 4 Hz.



- **At how many seconds do you expect the sound to be most quiet?**
- **At how many seconds do you expect the sound to be loudest?**
- **What is the period of this combined function?**
- **How many seconds do you expect between acoustic beats? Explain your answer.**

Standards Alignment

Practice Standards

MP4 > DOK 3

Model with mathematics. -- Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Content Standards

F-TF.5

Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

SBAC Claims

Mathematics Claim #1:
Concepts and Procedures. Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency.

Mathematics Claim #4:
Modeling and Data Analysis. Students can analyze complex, real-world scenarios and can use mathematical models to interpret and solve problems.

PARCC Sub-Claims

Sub-Claim A:
Major Content with Connections to Practices. The student solves problems involving the Major Content for her grade/course with connections to the Standards for Mathematical Practice.

Sub-Claim D:
Highlighted Practice MP.4 with Connections to Content: modeling/application. The student solves real-world problems with a degree of difficulty appropriate to the grade/course by applying knowledge and skills articulated in the standards for the current grade/course (or, for more complex problems, knowledge and skills articulated in the standards for previous grades/courses), engaging particularly in the Modeling practice, and where helpful making sense of problems and persevering to solve them (MP.1), reasoning abstractly and quantitatively (MP.2), using appropriate tools strategically (MP.5), looking for and making use of structure (MP.7), and/or looking for and expressing regularity in repeated reasoning (MP.8).

Scoring Rubric

4 Point Response:

The student response demonstrates:

- The ability to interpret periodic functions as models of sound waves;
- A strong ability to determine the frequency and period of a sine function;
- A strong ability to write an equation and sketch a graph of a sine function with a given frequency;
- A strong ability to interpret the frequency and period in terms of the pitch of a sound and interpret the amplitude in terms of the loudness of a sound.

The response demonstrates a high level of understanding. A level 4 response is characterized by:

- Correct determinations of the period and frequency for the given sine functions, with complete and correct work or explanations. Sketching the graphs of the functions to determine the frequency and period should be accepted as correct work;
- A correct equation for a sine function with a frequency of 440 Hz with complete and correct work or explanations;
- Correct sketches of sine functions that represent variations in the pitch and loudness of sounds;
- Correct equations and sketches of the graphs of sine functions with frequencies of 3 Hz and 4 Hz;
- The correct identification and interpretation of sections of the graph that represent constructive and destructive interference;
- A correct interpretation of the amplitude of the combined function in terms of loudness;
- A correct determination of the period of the combined function;
- A correct determination of the number of seconds between acoustic beats with a correct and complete explanation.

A sample level 4 response follows.

Part A: "For the general form of a sine curve, $y(x) = A\sin(Bx)$ and the period $= \frac{2\pi}{B}$. So the period for function 1 is $\frac{2\pi}{4\pi} = 0.5$ second, which means the frequency is 2 Hz (since frequency is the reciprocal of the period). Similarly, function 2 has a period of 1 second and the frequency is 1 Hz."

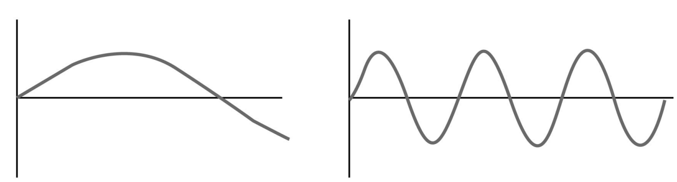
Part B: " As above, for the general form of a sine curve, $y(x) = A\sin(Bx)$ and the period $= \frac{2\pi}{B}$. Since the frequency is the reciprocal of the period, the frequency $= \frac{B}{2\pi}$. Setting $\frac{B}{2\pi} = 440$/sec leads to $B = (440)(2\pi)$, so a function that represents a sound with a frequency of 440 is $y(t) = \sin 880\pi t$."

Part C:

- Graph 1 shows a curve with a lower frequency than the given graph.
- Graph 2 shows a curve with a higher frequency than the given graph.
- Graph 3 shows a curve with a smaller amplitude than the given graph.
- Graph 4 shows a curve with a greater amplitude than the given graph.

(The graphs below show variations in frequency and amplitude. The reference graph is left out of the samples to make the graphs easier to interpret visually when printed in black and white.)

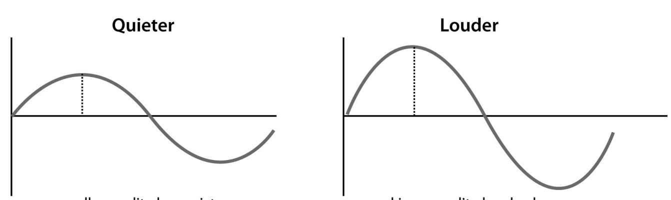
Variation in Frequency = Pitch



longer period = lower frequency = deeper pitch

shorter period = higher frequency = higher pitch

Variation in Amplitude = Loudness

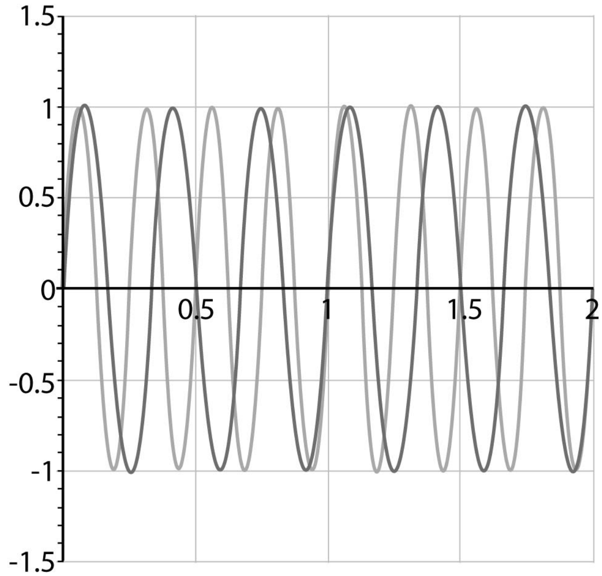


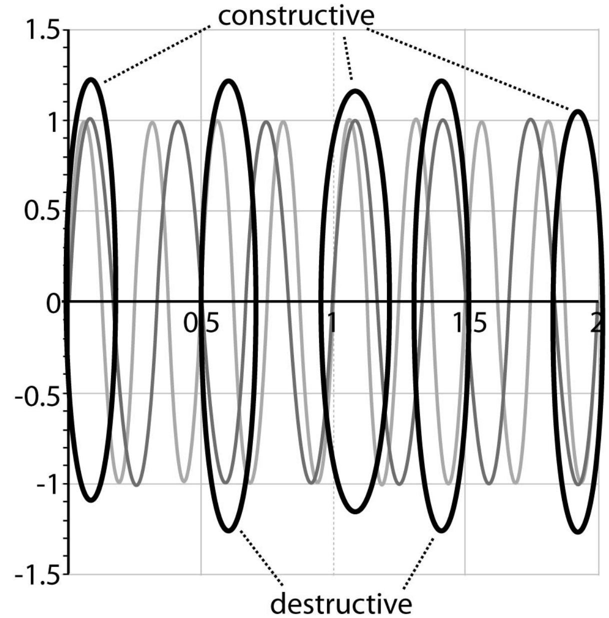
smaller amplitude = quieter

bigger amplitude = louder

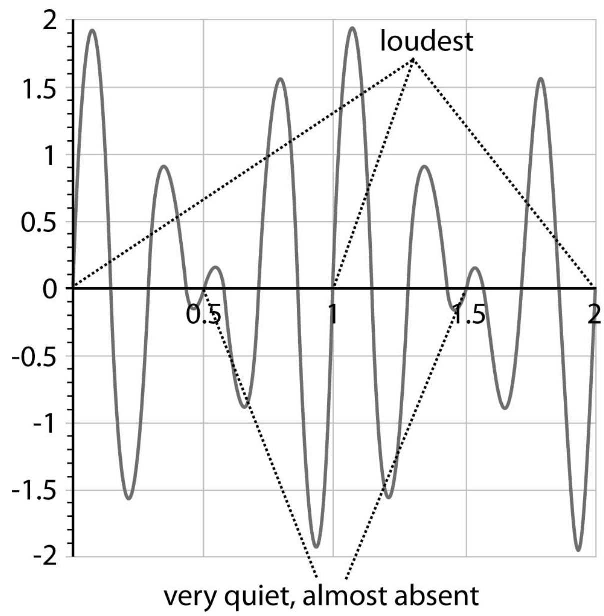
Part D:

$y(t) = \sin(6\pi t)$
$y(t) = \sin(8\pi t)$





Part E: "See the graphs for part D. The maximum amplitudes of the added function are where there is constructive interference and the minimum amplitudes are where there is destructive interference. The period of the function is 1 second, as can be seen by the amount of time between successive maximum or minimum amplitudes. When the function is at its maximum amplitude, you expect the sound to be loudest, and when it is as its minimum amplitude, you expect the sound to be very quiet, almost absent. A sound represented by this function sounds like a single note getting louder and softer with a period of 1 second."



3 Point Response: The response demonstrates a strong understanding, but the work contains minor errors. A level 3 response is characterized by: ● An ability to work with periodic functions as models of sound waves; ● A strong ability to determine the frequency and period of a sine function; ● A strong ability to write an equation and sketch a graph of a sine function with a given frequency, although the sketch may contain some minor inaccuracies; ● An ability to interpret the frequency and period in terms of the pitch of a sound and interpret amplitude in terms of the loudness of a sound, although the interpretations may be incomplete or contain some minor errors.
2 Point Response: The response demonstrates a basic but incomplete understanding. A level 2 response is characterized by: ● A basic ability to work with mathematical models of sound waves; ● A strong ability to determine the frequency and period of a sine function; ● A basic ability to write an equation and sketch a graph of a sine function with a given frequency, although the sketches of the graphs may contain errors; ● A weak ability to interpret the frequency and period in terms of the pitch of a sound and interpret amplitude in terms of the loudness of a sound. Responses in parts C, D, or E are vague, missing, or incorrect.
1 Point Response: The response demonstrates minimal understanding. A level 1 response is characterized by: ● A weak ability to interpret periodic functions as models of sound waves; ● A weak ability to determine the frequency and period of a sine function; ● A weak ability to write an equation and sketch a graph of a sine function with a given frequency; ● A weak ability to interpret the frequency and period in terms of the pitch of a sound and interpret amplitude in terms of the loudness of a sound. Responses to 4 or 5 of the parts are vague, missing, or incorrect.
0 Point Response: There is no response, or the response is off topic.

Discussion Questions

Use the following questions to help students struggling to access the problem:

1. What does the frequency of a wave tell us?

Possible response: *The frequency tells how many waves pass through a point in one second. You can think of Hertz as the number of waves per second.*

2. What does the period of a wave tell us?

Possible response: *The period is the amount of time it takes for one complete wave to pass through a point.*

3. What does the amplitude of a wave tell us?

Possible response: *The amplitude tells how high and low the wave goes. For a sound wave, the amplitude tells how loud the sound is. For an ocean wave, the amplitude tells how tall the wall of water is. For a light wave, the amplitude tells how bright the light is.*

Extension Activities

These activities require independent research and are DOK 4.

1. Research how “noise-canceling” headphones work.

   Sample: Since adding sounds can result in destructive interference, as we’ve seen in this task, noise-cancellation technology probably analyzes the waves of the sounds that are not desired, and produces a wave that causes only destructive interference. The best way to do this is to produce a wave that is the inverse of the original wave. If the original wave is represented by a sine function, the “noise-canceling” wave should be represented by the cosine function with the same argument.

2. Explore the mathematical basis of consonance and dissonance.

   Sample: Explore the graphs and equations of functions that represent consonant sounds, such as musical thirds, fifths, and octaves, and dissonant sounds, such as neutral seconds or other intervals. What relationship determines whether two sounds are consonant?

3. Investigate some physical and mathematical models of vibrating strings.

   Sample: Investigate models (both physical and mathematical) of vibrating strings with “stops” at different places (nodes).

4. Explore other physical phenomena that can be modeled using periodic functions, such as sine and cosine.

   Sample: Examples include the height of a pendulum, the tides, the length of daylight at a location over time, meandering streams, and the sunspot cycle.