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CCR Performance Tasks

Math III: Survey Results and Margin of Error

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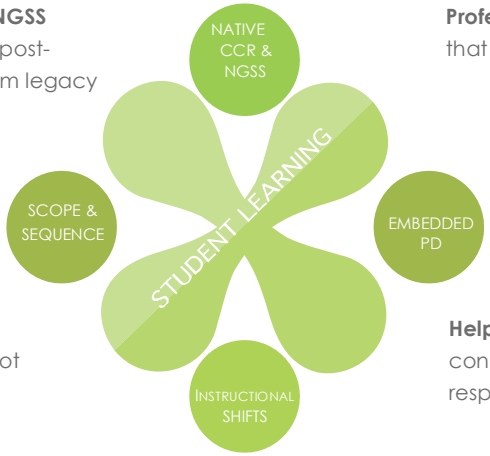

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CCR Performance Tasks

Math III: Survey Results and Margin of Error

Student Test Booklet

Name:

Math III: Survey Results and Margin of Error

Student Rubric

This problem is meant to test if you can:

- Determine a reasonable margin of error for a random sample of a given size;
- Use a random number generator and a spreadsheet to simulate random samples;
- Represent and describe data using basic statistics;
- Analyze the relevance of the margin of error in a given situation.

Your teacher will rate your answer as a level 4, 3, 2, 1, or 0. The descriptions below explain the type of answers expected at each level.

Level 4:
Your answer is correct and complete. Your answer includes:

- A complete description of the method you used to simulate the survey results;
- Simulated survey results;
- A correct and complete histogram that represents the simulated survey results;
- A correct and complete description of the data represented in the histogram, including shape, mean, and standard deviation;
- A correct determination of a reasonable margin of error;
- A correct analysis of the meaning of the survey results.

Level 3:
Your answer is mostly correct but one or two of your explanations are incomplete or you have made minor mistakes. Your answer includes:

- A description of the method you used to simulate the survey results, although the description may be incomplete;
- Simulated survey results;
- A histogram that represents the simulated survey results. The histogram may contain minor errors;
- A correct description of the data represented in the histogram, including shape, mean, and standard deviation;
- A correct determination of a reasonable margin of error;
- A correct but incomplete analysis of the meaning of the survey results.

Level 2:
You have answered some parts correctly, but some of your explanations are missing or weak and some parts contain mistakes. Your answer includes:

- A description of the method you used to simulate the survey results, although the description may be incomplete or contain errors;
- Simulated survey results, although you may have needed help in order to complete the simulations;
- A mostly correct histogram that represents the simulated survey results, although the graph may contain some minor errors;
- A partially correct description of the data represented in the histogram, including shape, mean, and standard deviation, although you may have made mistakes when calculating the mean and standard deviation;
- A determination of a margin of error, although the margin may be too large or too small;
- An analysis of the meaning of the survey results which contains errors, is vague, or is missing.

Level 1:
Your answers are incorrect. Your answer includes:

- A vague, missing, or incorrect description of the method you used to simulate the survey results;
- Simulated survey results, although you may have needed help in order to complete the simulations;
- A data display that is not a histogram, contains many errors, or does not appropriately represent the simulated survey results;
- A vague, missing, or incorrect description of the data represented in the data display;
- A missing or incorrect determination of a margin of error;
- A vague, missing, or incorrect analysis of the meaning of the survey results.

Level 0:
Your answer is not related to the question, the teacher cannot understand your answer, or you do not write anything.

Name:_____

Math III: Survey Results and Margin of Error

Complete all the tasks in the test booklet.

1 **In an election race between two candidates, a political pollster used a sample of 500 randomly selected likely voters to determine that one candidate was ahead of the other by 2 percentage points. The goal of this task is for you to determine what a reasonable margin of error is for a random sample of this size and to explain how this margin of error relates to the results reported by the pollster.**

A. Use a random-number generator and a spreadsheet to simulate 50 random samples of 500 voters. Describe the method you used to simulate random samples. Explain how the simulation represents the political polling situation.

Go On

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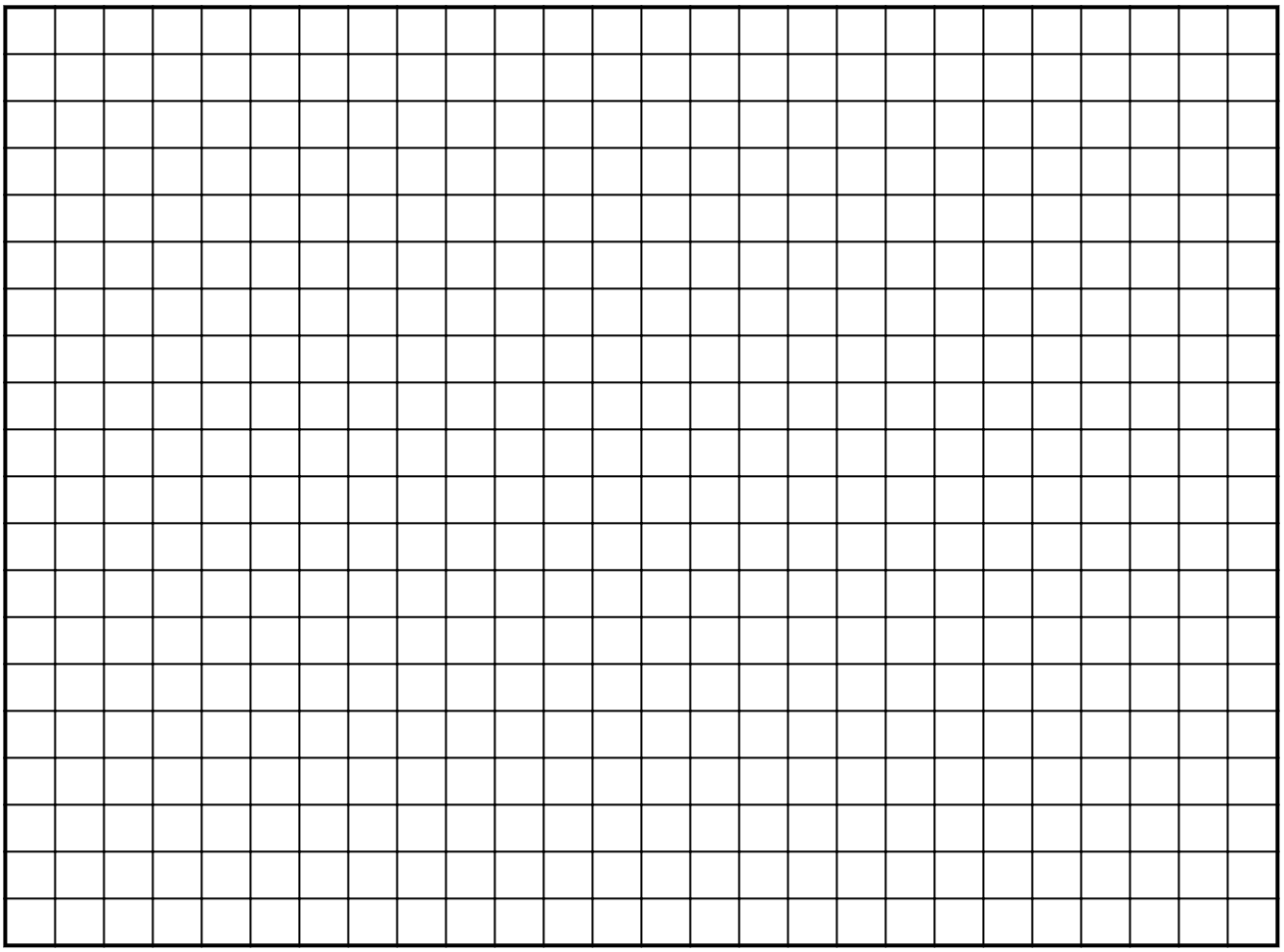
Name:_____

B. Create a histogram to represent the results of your simulations.



Go On

C. Describe the distribution of the simulated survey results with respect to shape, mean, and standard deviation. Use a calculator or your spreadsheet to determine the mean and standard deviation.

D. What is a reasonable margin of error for a sample size of 500? What does this mean about the political poll? Explain your reasoning.

CCR Performance Tasks

Math III: Survey Results and Margin of Error

Teacher Guide

About the Teacher Guide

This document contains support materials for “Math III: Survey Results and Margin of Error.”

This includes:

(a) The task
(b) The standards and depth of knowledge level of the task
(c) The scoring rubric
(d) Discussion questions
(e) Extension activities

These specifications have been included to help you connect the task to the Common Core content standards and the standards for mathematical practice. The rubric is designed to help you look for the development of mathematical practices in student work. It is also here to help you look for consistencies in student content errors that can help guide intervention and re-teach strategies.

Test Definition File

Item #	Correct Answer	Practice Standard	Content Standards
1	See Scoring Rubric	Mathematical Practice 4	S-IC.1, S-IC.4

SBAC Claims	PARCC Sub-Claims
1 and 4	A and D

Special Instructions

For a more real-world modeling or problem-solving experience, consider giving students only the introductory question: “A political poll used a sample of 500 randomly selected likely voters to determine that one candidate was ahead by 2 points. What is a reasonable margin of error for a random sample of this size?” The parts of the task offer scaffolding to help students through the steps of this problem. Advanced students should be able to break the problem down into parts themselves. Do not use the student rubric with the advanced modeling option.

Students need a spreadsheet and a random-number generator to complete part A. If these tools are not available, you can provide students with sample simulated survey results to complete the other parts of the task. Sample survey results are attached at the end of the teacher guide.

If some students have the tools available but do not know how to simulate the survey results, you may choose to provide the sample results for these students to complete the rest of the task. These responses should be considered level 2 or below.

Performance Task

In an election race between two candidates, a political pollster used a sample of 500 randomly selected likely voters to determine that one candidate was ahead of the other by 2 percentage points. The goal of this task is for you to determine what a reasonable margin of error is for a random sample of this size and to explain how this margin of error relates to the results reported by the pollster.

A. Use a random-number generator and a spreadsheet to simulate 50 random samples of 500 voters. Describe the method you used to simulate random samples. Explain how the simulation represents the political polling situation.

B. Create a histogram to represent the results of your simulations.

C. Describe the distribution of the simulated survey results with respect to shape, mean, and standard deviation. Use a calculator or your spreadsheet to determine the mean and standard deviation.

D. What is a reasonable margin of error for a sample size of 500? What does this mean about the political poll? Explain your reasoning.

Standards Alignment

Practice Standards

MP4 > DOK 3

Model with mathematics. -- Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Content Standards

S-IC.1

Understand statistics as a process for making inferences about population parameters based on a random sample from that population.

S-IC.4

Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.

SBAC Claims

Mathematics Claim #1:
Concepts and Procedures. Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency.

Mathematics Claim #4:
Modeling and Data Analysis. Students can analyze complex, real-world scenarios and can use mathematical models to interpret and solve problems.

PARCC Sub-Claims

Sub-Claim A:
Major Content with Connections to Practices. The student solves problems involving the Major Content for her grade/course with connections to the Standards for Mathematical Practice.

Sub-Claim D:
Highlighted Practice MP.4 with Connections to Content: modeling/application. The student solves real-world problems with a degree of difficulty appropriate to the grade/course by applying knowledge and skills articulated in the standards for the current grade/course (or, for more complex problems, knowledge and skills articulated in the standards for previous grades/courses), engaging particularly in the Modeling practice, and where helpful making sense of problems and persevering to solve them (MP.1), reasoning abstractly and quantitatively (MP.2), using appropriate tools strategically (MP.5), looking for and making use of structure (MP.7), and/or looking for and expressing regularity in repeated reasoning (MP.8).

Scoring Rubric

4 Point Response:

The student response demonstrates:

- A strong ability to use simulations to model survey results;
- A strong ability to represent and analyze data;
- A strong understanding of the margin of error.

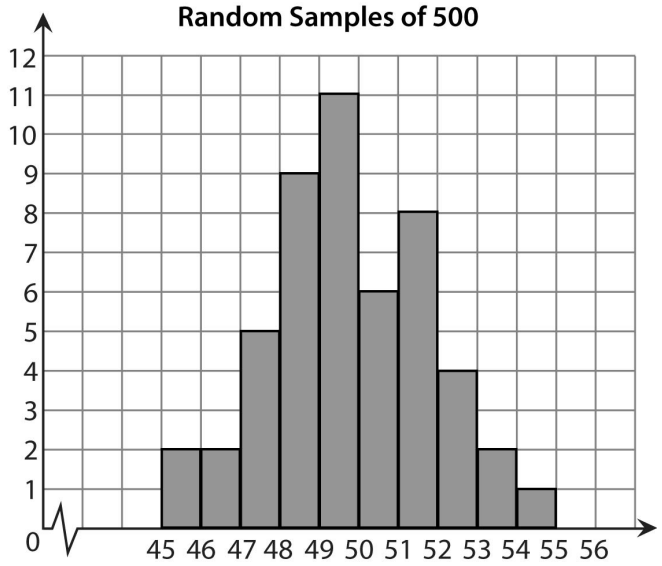
The response demonstrates a high level of understanding. A level 4 response is characterized by:

- A correct and complete description of the method used to simulate the survey;
- A correct and complete histogram representing the results of the simulations;
- A correct and complete description of the data represented in the histogram;
- A correct and complete interpretation of the results of the simulations with respect to the appropriate margin of error for the survey and the implications with respect to interpreting the meaning of the survey results.

A sample level 4 response follows.

Part A: "I used an online random number generator to generate a set of 500 integers from 0 to 1. I expect half of the numbers to be 1 and half to be 0. So the population parameter is 50%. I generated 50 sets of 500 0s and 1s. Then I copied the sets to a spreadsheet. I added all the numbers in each column, which tells me how many 1s I have in each sample. Then I divided the total number of 1s by 500 to get the frequency. I thought of it like this: a 1 represents a voter who says he will vote for one candidate, and a 0 represents a voter who says he will vote for the other candidate. I ignored undecided voters. If the sum of one of the columns is 242, that means that 242 out of 500 voters said they will vote for the first candidate."

Part B:



Part C: "The shape is mounded and approximately symmetrical. It looks pretty close to normally distributed. The mean is 49.89% and the standard deviation is 2.11."

Part D: "A reasonable margin of error is about +/- 4.5 percentage points. Two standard deviations from the mean includes about 95% of the data of a normal distribution. To be on the safe side, I rounded up 4.22 to 4.5. So if the voters are evenly split, the results of a 500-person random survey are within 4.5 points of 50% at least 95% of the time. This makes me think that a candidate being ahead by 2 points doesn't mean much because 2 points is within the margin of error. A candidate has to be ahead by about 5 points to have a significant lead. Or, if a series of polls all showed one candidate ahead by a few points, that also indicates a significant lead, because if the population parameter is 50% we expect one candidate to poll a little above 50% about half the time and a little below 50% about half the time."

3 Point Response:
The response demonstrates a strong understanding, but the work contains minor errors. A level 3 response is characterized by:

- A strong ability to use simulations to model survey results;
- A strong ability to represent and analyze data, although the graph may contain some minor errors;
- A strong understanding of the margin of error, although the explanation in part D may be incomplete.

2 Point Response:
The response demonstrates a basic but incomplete understanding. A level 2 response is characterized by:

- A basic ability to use simulations to model survey results. The student may need support from the teacher or peers to get started;
- A basic ability to represent and analyze data, although the graph may contain some minor errors, or the student may have made procedural errors using the spreadsheet or other tool to calculate mean or standard deviation;
- A basic understanding of the margin of error, although the interpretation and explanation in part D contains errors.

1 Point Response:
The response demonstrates minimal understanding. A level 1 response is characterized by:

- A lack of ability to use simulations to model survey results. The student cannot complete the simulations independently even after being shown how to do it;
- A weak ability to represent and analyze data;
- A weak understanding of the margin of error. The interpretation and explanation in part D may contain errors or be missing.

0 Point Response:
There is no response, or the response is off topic.

Discussion Questions

Use the following questions to help students struggling to access the problem:

1. What do these terms mean?
   a. Sample
   b. Sample size
   c. Sample statistic
   d. Population parameter
   e. Confidence level
   f. Margin of error
   g. Confidence Interval

Possible response:

a. *Sample: A subset of the population. In the case of the political poll, the sample is the likely voters who answered the question about which candidate they will vote for.*
b. *Sample size: The number of people (or objects) in the sample.*
c. *Sample statistic: A measure of an attribute of a population. In the case of the political poll, it is the fraction of the time that the people in the sample selected one candidate.*
d. *Population parameter: An attribute of a population. In the case of the political poll, it is the number of people who will actually vote for one candidate in the election. This is what we are trying to estimate by conducting the survey.*
e. *Margin of error: A measure of how close the sample statistic is likely to be to the population parameter. For example, if we flip a fair coin 10 times, we expect the coin to land with heads facing up 50% of the time. But we don't get 5 heads every time we do the experiment. Sometimes we get 3, 4, 6, or 7 heads, and occasionally we may get 0, 1, 2, 8, 9, or 10 heads. But if we do the experiment a large number of times, we see that most of the time we get close to 5. Suppose we get from 3 to 7 heads in 95% of the experiments. Then a reasonable margin of error is plus or minus 2. In other words, if we do the experiment 100 times, we expect the coin to land with heads facing up 3 to 7 times in 95 of those 100 experiments.*
f. *Confidence interval: This is the interval around the population parameter in which we expect our sample statistic to land. In the coin example above, the confidence interval is 3 to 7. When we do the coin toss experiment, we expect the coin to land heads up 3 to 7 times.*
g. *Confidence level: This is the fraction of the time that the sample statistic lands in the confidence interval (is within the margin of error of the population parameter). In the coin toss example above, the confidence level is 95%.*

2. How are the margin of error and confidence interval related?

Possible response: *The margin of error determines the confidence interval. If you have a large margin of error then you have a large confidence interval, which means you don't have a precise estimate of the population parameter. For example, if I say I predict gas prices will be $4 per gallon with a margin of error of plus or minus 30%, that means my confidence interval goes from 4 – 1.20 to 4 + 1.20, or $2.80 to $5.20.*

3. Is it more useful to have a large margin of error or a small margin of error?

Possible response: *It is generally more useful to have a small margin of error. This means you have a more precise prediction. Using the gas price example above, $2.80-$5.20 is a really large range. People might want to use the prediction to make estimates of how much money they will have to spend on gas. With a large confidence interval and margin of error, the prediction doesn't really help people make decisions.*

4. How is the confidence level related to the margin of error for the same-sized sample?

Possible response: *A greater margin of error generally allows for a greater confidence level. A good metaphor is an archery target. Having a greater margin of error is like having a bigger target. You hit the target more often if the target is big. The confidence level is like the fraction of the time you hit the target.*

5. How does changing the sample size affect the margin of error if the confidence level stays the same?

Possible response: *A larger sample size usually allows a smaller margin of error with the same confidence level. You essentially have more information and can use all that information to deliver more precise results.*

6. How can you use a random-number generator to simulate the results of a coin toss experiment?

Possible response: *You need half of the numbers generated to represent heads and half of the numbers to represent tails. You can use the integers 1 to 100 and count even integers as heads and odd integers as tails. Or you can use 0 and 1, and count 0 as heads and 1 as tails. Each random number generated represents the results of one coin toss. You can create a table or other data display to summarize the results of some number of tosses.*

Extension Activities

1. Look up other political polls or surveys of interest on the web. Simulate the surveys and determine an appropriate margin of error for a 95% confidence level.

2. Draw inferences from the survey data from multiple surveys. This activity can be DOK 4 if the students track a particular poll over time. The Pew Research Center has a wide variety of topics that it follows over time.

   A. Sample: Suppose you look at 6 political polls, and they all show one candidate a few points ahead of the other but always within the margin of error. Do these polls considered together paint a different picture than just one poll considered at a time?

   B. Sample: Suppose someone takes the same survey several times over a period of several months. How do the results change over time? What might this mean?

3. Simulate events with outcomes that are not equally likely.

   A. Sample: What does a distribution look like for a coin that lands with heads facing up 70% of the time?

   B. What types of survey results do you expect if a political candidate is favored by 55% of voters?

4. What is a reasonable confidence level for a published research paper? Is it different for different fields of research? This activity is DOK 4 because it involves students doing independent research and sharing findings with the class.

   A. Sample: Have students find and read published research papers. Look for margin of error, confidence level, and confidence intervals. Discuss the differences across fields.

Simulated Survey Results

Number of Respondents Who Selected One Candidate	Frequency of Selection of One Candidate
259	0.518
242	0.484
252	0.504
263	0.526
246	0.492
259	0.518
240	0.48
246	0.492
244	0.488
260	0.52
262	0.524
259	0.518
248	0.496
240	0.48
252	0.504
247	0.494
228	0.456
249	0.498
244	0.488
246	0.492
240	0.48
248	0.496
269	0.538
244	0.488
274	0.548
249	0.498
256	0.512
243	0.486
232	0.464
245	0.49
252	0.504
256	0.512
242	0.484
269	0.538
229	0.458
247	0.494
244	0.488
259	0.518
265	0.53
259	0.518
249	0.498
232	0.464
251	0.502
236	0.472
264	0.528
255	0.51
246	0.492
241	0.482
236	0.472
254	0.508