

Inspect

CCR Performance Task

Math I: Critique Reasoning and Solve Problems Using Inequalities

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CCR Performance Tasks

Math I: Critique Reasoning and Solve Problems Using Inequalities

Student Test Booklet

Name:

Math I: Critique Reasoning and Solve Problems Using Inequalities

Student Rubric

This problem is meant to test if you can:

- Critique a mathematical argument;
- Justify steps and recognize errors in someone's work;
- Make sense of and solve a problem using equations and inequalities.

Your teacher will rate your answer as a level 4, 3, 2, 1, or 0. The descriptions below explain the types of answers expected at each level.

Level 4:

Your answer is correct and complete.

Your answer includes:

- A correct and complete explanation of whether a conclusion is supported by the evidence and argument presented.
- A valid justification for each correct step in the solution to a linear inequality, or a valid suggestion for what should have been done at each incorrect step.
- A correct and complete solution to a problem, with all work shown.
- A correct interpretation of the results of calculations within the context of the problem.

Level 3:

Your answer is mostly correct but one or two of your explanations are incomplete or your work contains minor errors.

Your answer includes:

- A correct explanation of whether a conclusion is supported by the evidence and argument presented, but your explanation may be incomplete.
- A valid justification for each correct step in the solution to a linear inequality and valid suggestions for what should have been done at each incorrect step with at most one minor error.
- A correct solution to a problem, but your work may be incomplete or may contain one minor error.
- A correct interpretation of the results of calculations within the context of a problem.

Level 2:

You have answered only one part, or you have some errors in several parts.

Your answer may include:

- An incomplete explanation of whether a conclusion is supported by the evidence and argument presented, possibly with some minor errors.
- A valid justification for some correct steps in the solution of a linear inequality and valid suggestions for what should have been done at some incorrect steps.
- Work that shows a partially correct solution to a problem.
- An interpretation of the results of calculations within the context of a problem that is partially correct but is incomplete and contains some errors.

Level 1:

Your answers are incorrect.

Your answer may include:

- An incorrect explanation of whether a conclusion is supported by the evidence and argument presented.
- Invalid justification for correct steps in the solution to a linear inequality or invalid suggestions for what should have been done at incorrect steps.
- An incorrect solution to a problem, with minimal work shown and containing several minor or major errors.
- An incorrect interpretation of the results of calculations within the context of the problem.

Level 0:

Your answer is not related to the question, the teacher cannot understand your answer, or you do not write anything.

Name: _____

Math I: Critique Reasoning and Solve Problems Using Inequalities

Complete all the tasks in the test booklet.

- 1** For his birthday, Benny wants to invite friends to go bowling. There are two bowling alleys near his house, and the costs for birthday parties at both are shown below.

Gutter Guys	Strike Force
\$200 for up to 20 people \$10 for each person thereafter	\$80 set-up fee \$8 per person

Benny used this information to create the expressions below, where x is the number of people at the party.

- Gutter Guys: $200 + 10(x - 20)$
- Strike Force: $80 + 8x$

He then set up and solved the following inequality to determine the number of people for which Gutter Guys costs less:

$$\begin{aligned}\text{Step 1 : } & 200 + 10(x - 20) \leq 80 + 8x \\ \text{Step 2 : } & 200 + 10x - 20 \leq 80 + 8x \\ \text{Step 3 : } & 180 + 10x \leq 80 + 8x \\ \text{Step 4 : } & 2x \leq 100 \\ \text{Step 5 : } & x \leq 50\end{aligned}$$

Based on this result, Benny told his mother, "If we invite fewer than 50 people to the party, the Gutter Guys costs less. Since I'm inviting only 18 people, I think Gutter Guys is the better deal."

A. Is Benny's argument reasonable? Explain why or why not.

Name: _____

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B. Justify each step of Benny’s solution; that is, give a mathematical reason for why Benny can or cannot do what he did. If you think he made a mistake at any step, explain what he did wrong and indicate what he should have done instead.

Step 1: _____

Step 2: _____

Step 3: _____

Step 4: _____

Step 5: _____

C. Determine the number of people for which Gutter Guys is less expensive than Strike Force. Set up equations or inequalities based on the information provided and then solve the problem. Show all of your work.

Name: _____

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D. Interpret your solution from part C in terms of the context of the problem. When is Gutter Guys the better deal? When is Strike Force the better deal? Which bowling alley costs less for Benny's party?



CCR Performance Tasks

Math I: Critique Reasoning and Solve Problems Using Inequalities

Teacher Guide

About the Teacher Guide

This document contains support materials for “Math I: Critique Reasoning and Solve Problems Using Inequalities.”

This includes:

- (a) The task
- (b) The standards and depth of knowledge level of the task
- (c) The scoring rubric
- (d) Discussion questions
- (e) Extension activities

These specifications have been included to help you connect the task to the Common Core content standards and the standards for mathematical practice. The rubric is designed to help you look for the development of mathematical practices in student work. It is also here to help you look for consistencies in student content errors that can help guide intervention and re-teach strategies.

Test Definition File

Item #	Correct Answer	Practice Standard	Content Standards
1	See Scoring Rubric	Mathematical Practice 3	A-CED.1, A-CED.3, A-REI.1, A-REI.3

SBAC Claims	PARCC Sub-Claims
1 and 3	A and C

Performance Task

For his birthday, Benny wants to invite friends to go bowling. There are two bowling alleys near his house, and the costs for birthday parties at both are shown below.

Gutter Guys	Strike Force
\$200 for up to 20 people \$10 for each person thereafter	\$80 set-up fee \$8 per person

Benny used this information to create the expressions below, where x is the number of people at the party.

- Gutter Guys: $200 + 10(x - 20)$
- Strike Force: $80 + 8x$

He then set up and solved the following inequality to determine the number of people for which Gutter Guys costs less:

Step 1 : $200 + 10(x - 20) \leq 80 + 8x$
Step 2 : $200 + 10x - 20 \leq 80 + 8x$
Step 3 : $180 + 10x \leq 80 + 8x$
Step 4 : $2x \leq 100$
Step 5 : $x \leq 50$

Based on this result, Benny told his mother, “If we invite fewer than 50 people to the party, the Gutter Guys costs less. Since I’m inviting only 18 people, I think Gutter Guys is the better deal.”

- A. Is Benny’s argument reasonable? Explain why or why not.
- B. Justify each step of Benny’s solution; that is, give a mathematical reason for why Benny can or cannot do what he did. If you think he made a mistake at any step, explain what he did wrong and indicate what he should have done instead.
- C. Determine the number of people for which Gutter Guys is less expensive than Strike Force. Set up equations or inequalities based on the information provided and then solve the problem. Show all of your work.
- D. Interpret your solution from part C in terms of the context of the problem. When is Gutter Guys the better deal? When is Strike Force the better deal? Which bowling alley costs less for Benny’s party?

Standards Alignment

Practice Standards

MP3 > DOK 3

Construct viable arguments and critique the reasoning of others. -- Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and--if there is a flaw in an argument--explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen to or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Content Standards

A-CED.1

Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

A-CED.3

Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*

A-REI.1

Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

A-REI.3

Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Math I: Critique Reasoning and Solve Problems Using Inequalities

SBAC Claim

Mathematics Claim #1:

Concepts and Procedures. Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency.

Mathematics Claim #3:

Communicating Reasoning. Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.

PARCC Sub-Claims

Sub-Claim A:

Major Content with Connections to Practices. The student solves problems involving the Major Content for her grade/course with connections to the Standards for Mathematical Practice.

Sub-Claim C:

Highlighted Practices MP.3, 6 with Connections to Content: expressing mathematical reasoning. The student expresses grade/course-level appropriate mathematical reasoning by constructing viable arguments, critiquing the reasoning of others, and/or attending to precision when making mathematical statements.

Scoring Rubric

4 Point Response:

The response demonstrates a high level of understanding, including:

- A strong ability to critique an argument;
- A strong ability to justify correct algebraic steps in the solution to a linear equation or inequality;
- A strong ability to identify errors in the work of others;
- The ability to correctly set up and solve linear equations or inequalities;
- The capacity to correctly interpret the solution to a linear equation or inequality within the context of the problem.

A level 4 response is characterized by:

- A good critique of Benny's argument, including the model he created and the conclusions he drew. This should include noting that the expressions are correct for 20 or more people but the expression for Gutter Guys is not correct for fewer than 20 people, so the model works only if Benny invites more than 20 people. His conclusion is not supported by the model because the model works for 20 or more people and his conclusion is for 18 people;
- A valid justification for each correct step in Benny's solution, or a valid explanation for each incorrect step in Benny's solution, including the recognition that two equations or inequalities are needed to solve the problem completely;
- Two correctly formed equations or inequalities and a correctly worked solution based on those equations or inequalities;
- An accurate interpretation of the results from the equations or inequalities within the context of the problem.

A sample level 4 response follows.

Part A: "Benny's expressions work as long as you are talking about 20 or more people. But Gutter Guys charges a \$200 flat rate for up to 20 people, so the model does not always work. Benny's conclusion is about fewer than 20 people, but the model is not right for that many people. Also, Benny made mistakes in his calculations, so his conclusion is based on faulty mathematical reasoning. Ironically, it happens to be true that Gutter Guys is less expensive for 18 people, because $200 < 80 + 8(18) = 80 + 80 + 64 = 224$."

Part B: "Benny made several errors in his solution.

- In step 1, he had only one inequality, which works only when there are 20 or more people. He did not specify the two possible cases, $x < 20$ or $x \geq 20$. Because Gutter Guys charges \$200 for any number up to 20, Benny should have had a second inequality: $200 < 80 + 8x$.
- In step 2, Benny did not distribute the 10 correctly from step 1 to step 2. In step 2, the inequality should be $200 + 10x - 200 \leq 80 + 8x$, and then the 200 and -200 cancel each other out.
- In step 3, although Benny was using the wrong numbers here because of previous mistakes, he correctly subtracted.
- In step 4, Benny correctly subtracted $8x$ from both sides to leave $2x$ on the left, but when subtracting 180 from both sides, he should have gotten -100 on the right.
- In step 5, although Benny was using the wrong numbers here because of previous errors, he correctly divided both sides by 2."

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Part C, sample response 1: "A complete solution needs two inequalities, one for when $x \leq 20$ and one for when $x > 20$.

- When $x \leq 20$: $200 < 80 + 8x \rightarrow 120 < 8x \rightarrow 15 < x$.
- When $x > 20$: $200 + 10(x - 20) < 80 + 8x \rightarrow 200 + 10x - 200 < 80 + 8x \rightarrow 10x < 80 + 8x \rightarrow 2x < 80 \rightarrow x < 40$.

Gutter Guys is less expensive for $15 < x < 40$. To confirm, do some test cases:

- When $x = 45$, $200 + 10(45 - 20) = 450$ and $80 + 8(45) = 80 + 360 = 440$;
- When $x = 30$, $200 + 10(30 - 20) = 300$ and $80 + 8(30) = 80 + 240 = 320$;
- When $x = 18$, $80 + 8(18) = 80 + 144 = 224$, which is more than 200;
- When $x = 10$, $80 + 8(10) = 160$, which is less than 200."

Part C, sample response 2: "A full solution requires two cases: When $x > 20$, $200 + 10(x - 20) = 80 + 8x \rightarrow 200 + 10x - 200 = 80 + 8x \rightarrow 10x = 80 + 8x \rightarrow 2x = 80 \rightarrow x = 40$, and when $x \leq 20$, $200 = 80 + 8x \rightarrow 120 = 8x \rightarrow x = 15$. Then do some test cases:

- When $x = 30$, $200 + 10(30 - 20) = 300$ and $80 + 8(30) = 80 + 240 = 320$;
- When $x = 50$, $200 + 10(50 - 20) = 500$ and $80 + 8(50) = 80 + 400 = 480$;
- When $x = 10$, $80 + 8(10) = 160$, which is less than 200.

These results are interpreted in part D."

Part D, sample response 1: "Gutter Guys is the better deal for more than 15 people and fewer than 40 people. Strike Force is the better deal for more than 40 people or fewer than 15 people. Since Benny is inviting 18 people, and 18 is greater than 15 but less than 40, Gutter Guys costs less for Benny's party."

Part D, sample response 2: "The cost of Gutter Guys and Strike Force is equal for 15 people or 40 people. Between 15 and 40, however, Gutter Guys is less expensive. Since Benny is planning to have 18 people at his party, Gutter Guys is the better option. For 18 people, the cost at Gutter Guys is \$200, but the cost at Strike Force is \$224."

3 Point Response:

The response demonstrates a strong understanding, but the work contains minor errors. A level 3 response is characterized by:

- A strong ability to critique an argument, although the critique may be incomplete. This critique should include the recognition that two cases must be considered in this situation, for both more than 20 people and for 20 people or fewer;
- An ability to justify correct algebraic steps in the solution of a linear equation or inequality, possibly with one minor error;
- A strong ability to identify errors in the work of others, although one or two minor errors may be omitted;
- The ability to correctly set up and solve the applicable linear equations or inequalities, with at most one minor error;
- The capacity to correctly interpret the solution of a linear equation or inequality within the context of the problem, though the interpretation may be incomplete.

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2 Point Response:

The response demonstrates a basic but incomplete understanding. A level 2 response is characterized by:

- A basic ability to critique an argument although the critique is incomplete;
- A basic ability to justify correct algebraic steps in the solution to a linear equation or inequality, possibly with several minor errors;
- A basic ability to identify errors in the work of others, though the rationale may not be articulated, possible corrections may not be suggested, and some errors may be omitted;
- The ability to correctly set up but not necessarily solve a linear equation or inequality, possibly making more than one minor error or one major error. For this level, it is possible that only one of two cases is considered, and only one equation or inequality is set up;
- A basic capacity to interpret the solution to a linear equation or inequality within the context of the problem, though the interpretation may be incomplete or contain errors.

1 Point Response:

The response demonstrates minimal understanding. A level 1 response is characterized by:

- A weak ability to critique an argument. The critique may contain misstatements or be missing;
- A weak ability to justify correct algebraic steps in the solution to a linear equation or inequality;
- An inability to identify errors in the work of others, to explain the errors, or to suggest corrections;
- A rudimentary ability to set up but not necessarily solve a linear equation or inequality, with errors in both the setup and solution;
- An inability to interpret the solution of a linear equation or inequality within the context of the problem.

0 Point Response:

There is no response, or the response is off topic.

Discussion Questions

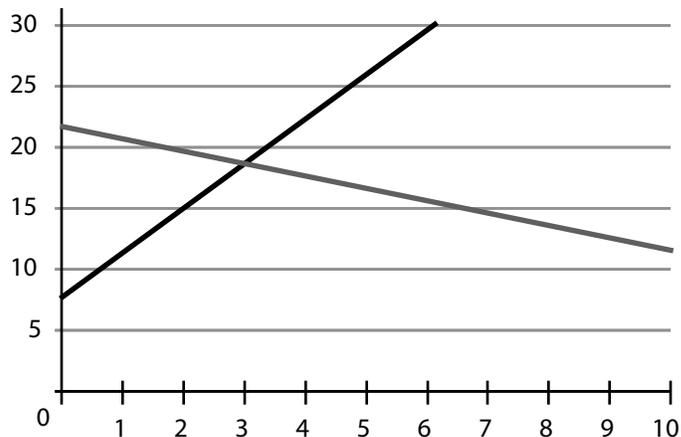
Use the following questions to stimulate discussion:

1. When solving an algebraic equation, what are some important things to keep in mind?

Possible Response: *The equals sign is a balance point, which means that whatever you do to one side, you must do to the other. For example, if you subtract $2x$ from the left, you must subtract $2x$ from the right; if you divide the right side by 6, you need to also divide the left side by 6. In addition, solving a linear equation means finding a value of x for which the equation is true. Therefore, remember to isolate x on one side of the equation, and get all of the numbers on the other side.*

2. After solving a linear equation, what are some ways that you could check to see if your answer is correct?

Possible Response: *One method for checking the solution is to test the value in the original equation. For instance, if you find that $x = 3$ is a solution for the equation $4x + 7 = 22 - x$, then check to see if $4(3) + 7 = 22 - 3$ is true. Another method is to graph both sides of the linear equation to see where they intersect. For the example above, graph the two lines $y = 4x + 7$ and $y = 22 - x$. The two lines intersect when $x = 3$, so that verifies the solution.*



3. Is solving inequalities different than solving equations?

Possible Response: *Mostly it is the same, but you may need to check more than one point to make sure your answer is correct. For example, if I solve $4x + 7 < 22 - x$, I get $x < 3$. I might try out $x = 0$ and $x = 5$ to make sure the relation is true.*

Extension Activities

1. Developing a further understanding of linear equations.

A. Create multiple representations for the same situation, such as equations, graphs, and tables of values, to see how they are related.

Sample: Consider the linear equation $4x + 7 = 22 - x$. The equation can be solved by adding x to both sides and subtracting 7 from both sides to give $5x = 15$, which yields $x = 3$. It can be powerful for students to then graph $y = 4x + 7$ and $y = 22 - x$ and see that they intersect at $x = 3$; this helps to develop conceptual understanding about what it means to solve an equation. Finally, a table of values shows yet another connection, because it reinforces the idea that x is a variable and can take on numerous values. The table of values below shows that the two sides of the equation are equal when $x = 3$.

x	$4x + 7$	$22 - x$	Are they equal?
1	$4(1) + 7 = 11$	$22 - 1 = 21$	No
2	$4(2) + 7 = 15$	$22 - 2 = 20$	No
3	$4(3) + 7 = \mathbf{19}$	$22 - 3 = \mathbf{19}$	Yes
4	$4(4) + 7 = 23$	$22 - 4 = 18$	No

2. Developing a deeper understanding of the common mistakes that are made when solving linear equations. As a class, develop a list of common mistakes, along with explanations for why they are incorrect and examples of what is correct.

A. Discuss why $4x + 7 = 22 - x$ does not simplify to $3x + 7 = 22$.

Sample: It is common for students to get rid of a negative value from one side of an equation by subtracting that same value from the other side. One way to show these are not equivalent is to solve $3x + 7 = 22$ to yield $x = 5$, and then show that it does not yield a true result when substituted into the original equation. Another option is to graph both equations; on one set of axes, graph $y = 4x + 7$ and $y = 22 - x$, and point out that they intersect at $x = 3$; on another set of axes, graph $y = 3x + 7$ and $y = 22$, and note that they intersect at $x = 5$. If the equations were equivalent, they would intersect at the same x -value.

B. Discuss why $3(x - 2) = 4x + 11$ does not simplify to $3x - 2 = 4x + 11$.

Sample: It is common for students to distribute a coefficient to only the first term within parentheses. To give a concrete example of why this is incorrect, remind students of basic multiplication. The multiplication problem 3×12 can be thought of as $3(10 + 2)$, which is analogous to the equation above, except that it uses 10 instead of x . It would be wrong to compute $3(10 + 2) = 3 \times 10 + 2 = 32$, and students know that $3 \times 12 = 36$. Instead, the 3 needs to be distributed as $3(10 + 2) = 3(10) + 3(2) = 30 + 6 = 36$.

C. If students are ready to study quadratic equations, you could further extend this by discussing the multiplication of two binomials, such as $(x - 3)(x + 2)$. By analogy, you could show that $13 \times 12 = (10 + 3)(10 + 2) = 10(10) + 10(2) + 3(10) + 3(2) = 156$. It's important to emphasize that there are four intermediate multiplications that occur when completing this computation with numbers, and the same is true when multiplying binomials.