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CCR Performance Task

Math II: Analyze a Map

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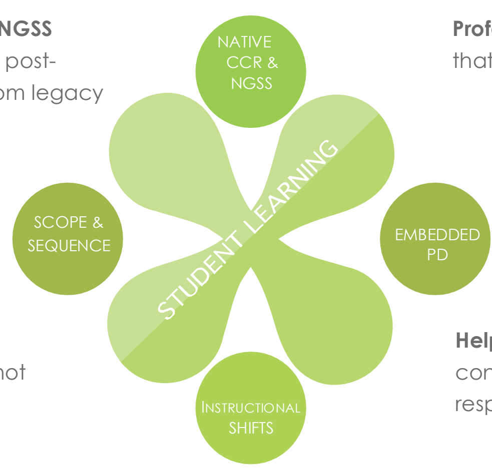

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CCR Performance Tasks

Math II: Analyze a Map

Student Test Booklet

Name:

Student Rubric

This problem is meant to test if you can:
• Apply geometric reasoning to a real-world situation;
• Determine the areas of triangular and trapezoidal regions;
• Determine the lengths of line segments;
• Determine angle measures.

Your teacher will rate your answer as a level 4, 3, 2, 1, or 0. The descriptions below explain the types of answers expected at each level.

Level 4:
Your answer is correct and complete.
Your answer includes:
• Correct calculations of the areas for six regions with all work shown or explained.
• Correct calculations of the lengths for six line segments with all work shown or explained.
• Correct calculations of the angle measures for the six angles at a point of intersection, including at least one application of trigonometric ratios, with all work shown or explained.

Level 3:
Your strategy to solve the problem is correct but you have made some minor errors in your calculations.
Your answer includes:
• Correct strategies to calculate the areas for six regions with minor errors in some of the calculations and all work shown or explained.
• Correct strategies to calculate the lengths for six line segments with minor errors in some of the calculations with all work shown or explained.
• Correct strategies to calculate the angle measures for six angles at a point of intersection with minor errors in some of the calculations with all work shown or explained.

Level 2:
You have shown some basic understanding of the problem but you are not able to solve it completely, or your solution contains several errors.
Your answer may include:
• Correct strategies to calculate the areas of at least three of six regions, with work shown or explained.
• Correct strategies to calculate the lengths for at least three of six line segments with work shown or explained.
• Correct strategies to calculate the angle measures for at least three of six angles at a point of intersection, including at least one application of trigonometric ratios, with work shown or explained.

Level 1:
Your answers are incorrect.
Your answer may include:
• Incorrect calculations for the areas of six regions, with several major errors.
• Incorrect calculations of the lengths of six line segments, with several major errors.
• Incorrect calculations of the angle measures for at least four of six angles at the point of intersection, with several major errors.

Level 0:
Your answer is not related to the question, the teacher cannot understand your answer, or you do not write anything.

Name:______________________________

Math II: Analyze a Map

Complete all the tasks in the test booklet.

1 The map on the last page of this test booklet shows Menta Park.

Menta Park is a large rectangular space in the downtown area of Velo City. A community group petitioned the city to add walking paths in Menta Park. After analyzing foot traffic in the area, the city has decided to add three paths. The paths and the locations of the center lines of the paths are described below.

- The center lines of all three paths intersect at one point within the park. A special gazebo is to be placed at the intersection point.
- The first path connects the northwest and southeast corners of the park. The center line of this planned path runs from the corner of the park at the intersection of Jefferson Drive and Second Avenue to the corner of the park at the intersection of Madison Boulevard and First Avenue.
- The second path connects the northeast corner of the park with a point on the south side of the park along First Avenue. The center line of the planned path runs from the corner of the park at the intersection of Madison Boulevard and Second Avenue to a point along the north side of First Avenue exactly halfway between Jefferson Drive and Madison Boulevard. The center line of the path forms a 45° angle with First Avenue.
- The third path is parallel to First Avenue and connects Seri Alley to Fisk Alley. The center line of this planned path intersects Jefferson Drive and Madison Boulevard at points exactly one-third of the distance from First Avenue to Second Avenue.

The distance between Madison Boulevard and Jefferson Drive along the north side of Menta Park is 1,200 feet.

A. The three planned walking paths divide the park into six regions. Draw the center line of each of these paths on the map. Label the corners and intersections so that you can identify all 6 regions of the park. Determine the area of each region, in square feet. Show your work and label your answers clearly.

To answer, draw directly on the map and show your calculations in the blank area provided on page 2.

B. Six path segments connect the point where the center lines of the paths intersect to points on the outer edge of the park. What is the length of the center line of each path segment?

Show your work in the blank area on page 3. Indicate the length of each path segment on the map.

C. Six angles are formed at the point where the center lines of the paths intersect. The architect who is designing the gazebo for this location needs to know the measures of these six angles. Find the measures of the six angles.

Show your work in the blank area on page 4. Indicate the angle measures on the map.

Name:______________________________

Math II: Analyze a Map

Part A

Name:________________________________

Math II: Analyze a Map

Part B

Name:______________________________

Part C

STOP

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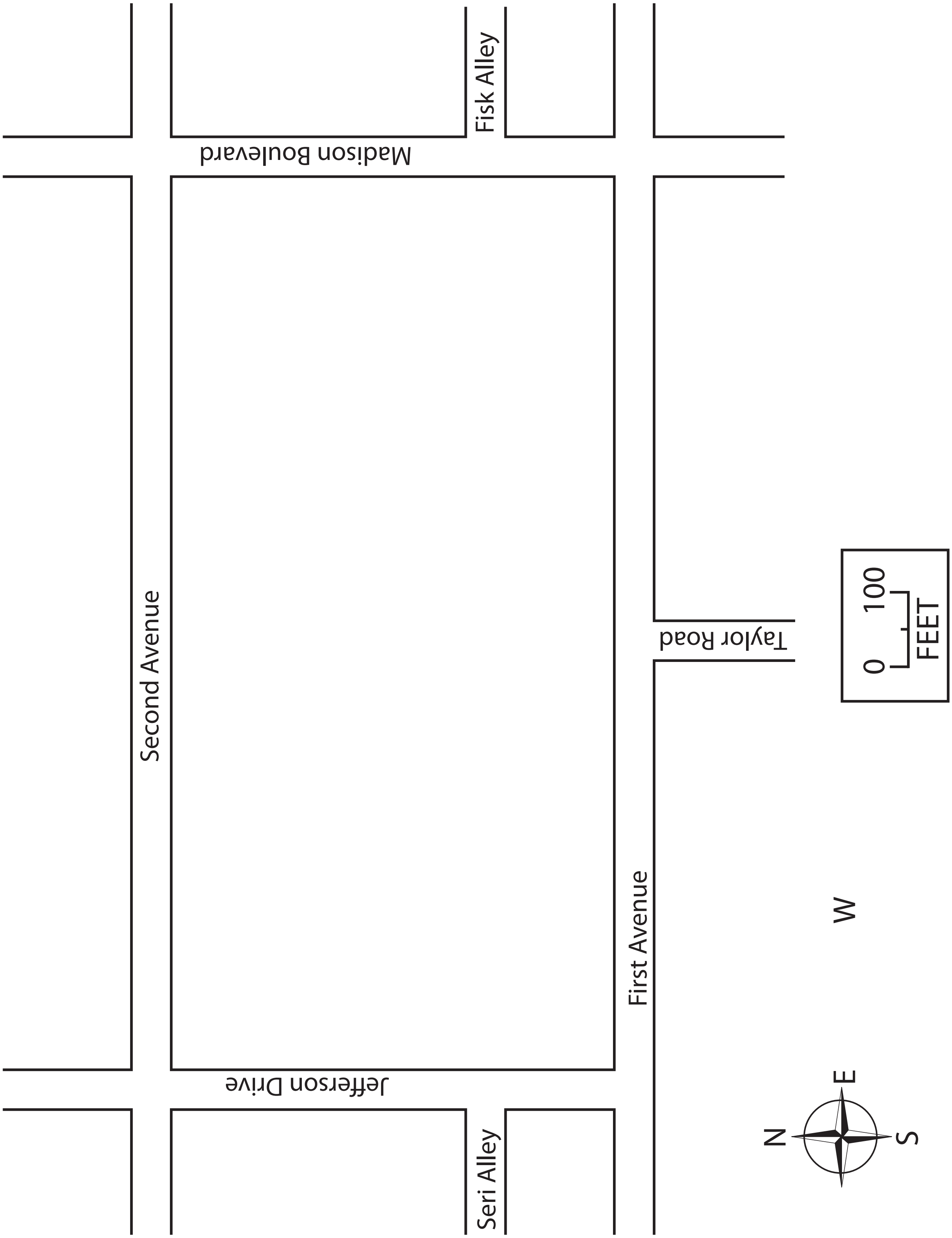
Name: ____

Math II: Analyze a Map



CCR Performance Tasks

Math II: Analyze a Map

Teacher Guide

About the Teacher Guide

This document contains support materials for “Math II: Analyze a Map.”
This includes:

(a) The task
(b) The standards and depth of knowledge level of the task
(c) The scoring rubric
(d) Discussion questions
(e) Extension activities

These specifications have been included to help you connect the task to the Common Core content standards and the standards for mathematical practice. The rubric is designed to help you look for the development of mathematical practices in student work. It is also here to help you look for consistencies in student content errors that can help guide intervention and re-teach strategies.

Test Definition File

Item #	Correct Answer	Practice Standard	Content Standards
1	See Scoring Rubric	Mathematical Practice 4	G-SRT.8

SBAC Claims	PARCC Sub-Claims
1 and 4	A and D

Performance Task

The map on the last page of this test booklet shows Menta Park.

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- The center lines of all three paths intersect at one point within the park. A special gazebo is to be placed at the intersection point.
- The first path connects the northwest and southeast corners of the park. The center line of this planned path runs from the corner of the park at the intersection of Jefferson Drive and Second Avenue to the corner of the park at the intersection of Madison Boulevard and First Avenue.
- The second path connects the northeast corner of the park with a point on the south side of the park along First Avenue. The center line of the planned path runs from the corner of the park at the intersection of Madison Boulevard and Second Avenue to a point along the north side of First Avenue exactly halfway between Jefferson Drive and Madison Boulevard. The center line of the path forms a 45° angle with First Avenue.
- The third path is parallel to First Avenue and connects Seri Alley to Fisk Alley. The center line of this planned path intersects Jefferson Drive and Madison Boulevard at points exactly one-third of the distance from First Avenue to Second Avenue.

The distance between Madison Boulevard and Jefferson Drive along the north side of Menta Park is 1,200 feet.

A. The three planned walking paths divide the park into six regions. Draw the center line of each of these paths on the map. Label the corners and intersections so that you can identify all 6 regions of the park. Determine the area of each region, in square feet. Show your work and label your answers clearly.

To answer, draw directly on the map and show your calculations in the blank area provided on page 2.

B. Six path segments connect the point where the center lines of the paths intersect to points on the outer edge of the park. What is the length of the center line of each path segment?

Show your work in the blank area on page 3. Indicate the length of each path segment on the map.

C. Six angles are formed at the point where the center lines of the paths intersect. The architect who is designing the gazebo for this location needs to know the measures of these six angles. Find the measures of the six angles.

Show your work in the blank area on page 4. Indicate the angle measures on the map.

Standards Alignment

Practice Standards

MP4 > DOK 3

Model with mathematics. -- Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Content Standard

G-SRT.8

Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

SBAC Claims

Mathematics Claim #1:
Concepts and Procedures. Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency.

Mathematics Claim #4:
Modeling and Data Analysis. Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems.

PARCC Sub-Claims

Sub-Claim A:
Major Content with Connections to Practices. The student solves problems involving the Major Content for her grade/course with connections to the Standards for Mathematical Practice.

Sub-Claim D:
Highlighted Practice MP.4 with Connections to Content: modeling/application. The student solves real-world problems with a degree of difficulty appropriate to the grade/course by applying knowledge and skills articulated in the standards for the current grade/course (or, for more complex problems, knowledge and skills articulated in the standards for previous grades/courses), engaging particularly in the Modeling practice, and where helpful making sense of problems and persevering to solve them (MP.1), reasoning abstractly and quantitatively (MP.2), using appropriate tools strategically (MP.5), looking for and making use of structure (MP.7), and/or looking for and expressing regularity in repeated reasoning (MP.8).

Scoring Rubric

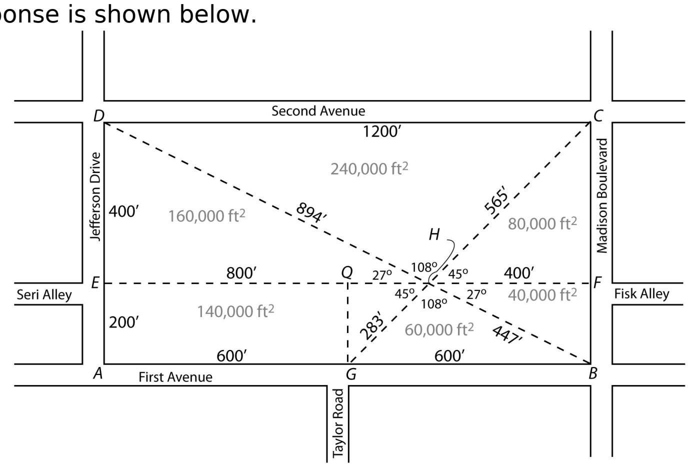
4 Point Response:
The response demonstrates a high level of understanding, including:

- An ability to determine the area of triangular and trapezoidal regions using an area formula or other geometric reasoning;
- An ability to determine segment lengths using the Pythagorean theorem or other geometric reasoning;
- An ability to use trigonometric ratios or geometric theorems to determine angle measures.

A level 4 response is characterized by:

- A correct calculation of the area for each of the six regions;
- A correct calculation of the length for each of the path center line segments;
- A correct calculation of the angle measures for the six angles at the point of intersection, including at least one application of trigonometric ratios, with all work shown.

A sample 4 point response is shown below.



Part A, sample response 1: "Point *G* divides *AB* into two equal parts, so *AG* = *GB* = 600 feet. The path between Seri and Fisk divides the park into two rectangles, and the top rectangle is twice the size of the bottom rectangle. Hence, *AE* = 200 ft and *ED* = 400 ft.
Triangle *HCF* is an isosceles right triangle, so *FC* = 400 ft = *HF*. Further, the total length of *EF* is 1200 ft, so *EH* = 800 ft because *HF* = 400 ft. From these numbers, the area of each region can be found:

- *HGB* = $\frac{1}{2}$ x 600 x 200 = 60,000 square feet
- *HFB* = $\frac{1}{2}$ x 200 x 400 = 40,000 square feet
- *HFC* = $\frac{1}{2}$ x 400 x 400 = 80,000 square feet
- *HCD* = $\frac{1}{2}$ x 1,200 x 400 = 240,000 square feet
- *HDE* = $\frac{1}{2}$ x 400 x 800 = 160,000 square feet
- *HEAG* = the total area of the park minus the sum of the areas above = 1200 x 600 – (60,000 + 40,000 + 80,000 + 240,000 + 160,000) = 140,000 square feet."

Part A, sample response 2: "The sidewalk between Seri and Fisk divides the park into two rectangles, and the top rectangle is twice the size of the bottom rectangle. Hence, *BF* = 200 ft and *FC* = 400 ft. Triangle *HCF* is an isosceles right triangle. Its area is $\frac{1}{2}$ x 400 x 400 = 80,000 square feet. Trapezoid *HEAG* consists of rectangle *AGQE* and isosceles right triangle *HGQ*, which is similar to triangle *HCF.* The area of rectangle *AGQE* is 200 x 600 = 120,000 square feet, and the area of triangle *HGQ* is 20,000 square feet, because it is $\frac{1}{4}$ the area of triangle *HCF*, so the area of trapezoid *HEAG* is 140,000 square feet. The height of triangle *HCD* is equal to $\frac{2}{3}$ the height of rectangle *ABCD*, and its base is equal to the length of rectangle *ABCD,* so its area is $\frac{1}{2} \times \frac{2}{3} = \frac{1}{3}$ the area of rectangle *ABCD*. Since the area of rectangle *ABCD* is 1,200 x 600 = 720,000 square feet, the area of triangle *HCD* is 240,000 square feet. Further, the area of triangle *HGB* is $\frac{1}{4}$ the area of triangle *HCD*, because triangle *HGB* is similar to triangle *HCD* but its dimensions are $\frac{1}{2}$ the dimensions of *HCD*. Hence, the area of triangle *HBG* is 60,000 square feet. The area of triangle *HBF* is $\frac{1}{2} \times 400 \times 200 = 40{,}000$ square feet. Triangle *HDE* is similar to triangle *HBF* but the area is 4 times as large, so its area is 160,000 square feet."

Part B: "*EF* is 1,200 feet because it is parallel to the north edge of the park along Second Avenue. From above, *FC* = 400 ft and triangle *HCF* is an isosceles right triangle, so *HF* = 400 ft. Consequently, *EH* = 1,200 - 400 = 800 feet. Segment *HC* is the hypotenuse of a 45-45-90 triangle, so its length is $\sqrt{(2)} \times 400 \approx 565$ feet. Triangle *HFC* is similar to triangle *HGQ*, so its hypotenuse is $\frac{1}{2}$ the length of *HC*. Hence, *HG* ≈ 283 feet. Finally, *BD* is the hypotenuse of triangle *ABD*, so its length is $\sqrt{(1200^2 + 600^2)} = 1341.64$ feet. Since triangles *DEH* and *DAB* are similar, point *H* occurs $\frac{2}{3}$ of the way from *D* to *B*, so *DH* = 2/3 x 1341.64 ≈ 894 feet, and $HB = \frac{1}{3} \times 1341.64 \approx 447$ feet."

Part C: "In the statement of the problem, it was given that $\angle HGB = 45°$. Segment *CG* is a transversal that cuts parallel lines *EF* and *AB*, so $m\angle CHF = m\angle HGB = 45°$. Then, because $\angle CHF$ and $\angle EHG$ are vertical angles, $m\angle CHF = m\angle EHG = 45°$. In part B, it was found that *HE* = 800 feet and *DE* = 400 feet. So $\tan(\angle DHE) = \frac{400}{800} = \frac{1}{2}$, which means that $m\angle DHE = \arctan(\frac{1}{2}) \approx 27°$. Because $\angle DHE$ and $\angle FHB$ are vertical angles, $m\angle FHB = m\angle DHE \approx 27°$. The sum $m\angle DHE + m\angle DHC + m\angle CHF = 180°$, which implies that $m\angle DHC = 180 – 45 – 27 = 108°$. Because $\angle DHC$ and $\angle GHB$ are vertical angles, $m\angle DHC = m\angle GHB = 108°$."

Note: Once the measures of all line segments were found in part B, any of the angle measures can be found using trigonometry. The solution above represents only one of many possible solution strategies.

3 Point Response:

The response demonstrates a strong understanding but the work contains minor errors. A level 3 response is characterized by:

- An ability to determine the area of triangular and trapezoidal regions using an area formula or other geometric reasoning, though with minor errors or incomplete explanations;
- An ability to determine segment lengths using the Pythagorean theorem or other geometric reasoning, though with minor errors or incomplete explanations;
- An ability to use trigonometric ratios or geometric theorems to determine angle measures, though with minor errors or incomplete explanations.

2 Point Response:

The response demonstrates a basic but incomplete understanding. A level 2 response is characterized by:

- A basic ability to determine the area of triangular and trapezoidal regions using an area formula or other geometric reasoning, though with minor errors or incomplete explanations;
- A basic ability to determine segment lengths using the Pythagorean theorem or other geometric reasoning, though with errors or incomplete explanations;
- A weak ability to use trigonometric ratios or geometric theorems to determine angle measures, though with errors or incomplete explanations.

1 Point Response:

The response demonstrates minimal understanding. A level 1 response is characterized by:

- A basic ability to determine the area of triangular and trapezoidal regions, with minor errors and incomplete explanations;
- A weak ability to determine segment lengths, with major errors and incomplete explanations. Work for part B may be missing;
- A weak ability to determine angle measures, with major errors and incomplete explanations. Work for part C may be missing.

0 Point Response:

There is no response, or the response is off topic.

Discussion Questions

Use the following questions to stimulate discussion:

1. How can you determine the area of a region?

Possible Response: *If the region is a standard shape (triangle, rectangle, parallelogram) and the base and height are known, you can use an area formula. An alternative is to compute the area of a larger region surrounding the figure, and then subtract the area of the regions surrounding the shape. (For instance, draw a rectangle around a quadrilateral, then subtract the areas of the triangles that surround the quadrilateral.) It may also be possible to decompose a large, irregular shape into smaller, more manageable regions, such as dividing a pentagon into three triangles, then determining the area of the smaller shapes and adding them together.*

2. What are some methods for determining the length of a segment?

Possible Response: *Obviously, segments can be measured. But for calculation purposes, the Pythagorean theorem or trigonometric ratios can be used. Which one is more appropriate depends upon the information you already have. The Pythagorean theorem can only be used for right triangles when two of the side lengths are known. Trigonometry, however, can be used for any type of triangle as long as at least two pieces of information about the triangle are known.*

3. How can trigonometric relationships be useful when solving geometry problems?

Possible Response: *Trigonometry can be used to determine angle measures when side lengths are known or to determine side lengths when angle measures are known. Knowing two pieces of information about a triangle is usually sufficient to determine the other four angle measures or side lengths.*

Extension Activities

1. Develop a deeper understanding of when various methods can be used for determining side lengths.

   A. Generate a list of situations when the Pythagorean theorem or trigonometric ratios can and cannot be used.
   Sample: The Pythagorean theorem cannot be used if the triangle does not contain a right angle. Trigonometric ratios can be used for any triangle—for instance, they can be used to find the angle measures of a triangle with sides of 8 cm, 12 cm, and 15 cm. For shapes with more than three sides, you can divide the shape into triangles and then use trigonometric ratios.

2. Develop a further understanding about trigonometric ratios.

In addition to the three standard trigonometric ratios (sine, cosine, and tangent), students should be exposed to the Law of Sines and the Law of Cosines.

Sample:
- Give students a triangle with specific values for the side lengths and angle measures, such as $a = 200$, $b = 173$, $c = 100$, $m\angle A=90^{o}$, $m\angle B=60^{o}$, $m\angle C=30^{o}$.
- Direct students to compute $\frac{\sin A}{a}$, $\frac{\sin B}{b}$, and $\frac{\sin C}{c}$, and compare the results. (They should find that all three ratios have the same value.)
- Give students a general triangle with side lengths a, b, and c, and angles A, B and C. Draw an altitude of length k from B to AC. This creates two right triangles such that $\sin A=\frac{k}{c}$ and $\sin C=\frac{k}{a}$. Solving both for k and setting them equal gives $c \sin A=a \sin C$, or $\frac{\sin A}{a}=\frac{\sin C}{c}$, which is exactly what they found with specific values above. Repeating this with altitudes from A and C will show that $\frac{\sin A}{a} = \frac{\sin B}{b}= \frac{\sin C}{c}$, which is the Law of Sines.

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