Inspect

CCR Performance Task

Math Grade 6: Solve Problems with Fractions

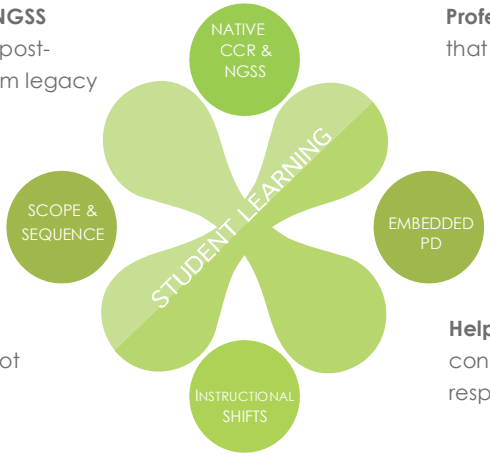
Inspect offers the following assessment products:

Content Bank for English/Language Arts and Math Grades 2 – High School	▪ More than 36,000 items ▪ More 1500 complex texts, including authentic permissioned texts ▪ Includes Literacy in History, Social Science, Science, and Technical Subjects
Quick Checks for English/Language Arts and Math Grades 2 – High School	▪ Fixed-form assessments with five to seven items including constructed response ▪ Key instructional concepts embedded in standards (clusters for Math, staircase of text complexity for ELA)
Focused Interim Assessments for English/Language Arts and Math Grades 3 – High School	▪ Prebuilt assessments with up to 15 items that focus on groups of related standards within a Claim or domain ▪ More focused than summative assessments ▪ Flexible and customizable ▪ Mirrors SBAC IAB blueprints
NGSS Formative Assessments Grades 5 – High School	▪ Prebuilt assessments with items linked to experimental contexts that assess the three dimensions of science learning ▪ Flexible and customizable ▪ Addresses the California Course Models and NGSS Bundles
Observational Tasks for English/Language Arts and Math Grades K - 1	▪ Developmentally appropriate for individual students and small groups

Inspect Assessment Content is available through a variety of assessment administration and data analysis platforms.

Inspect assessment content offers these benefits:

Native college- and career-ready and NGSS content prepares students to meet their post-secondary goals. Content re-aligned from legacy standards cannot do this.



Professional development embedded within content that

- shows the relationship between specific skills and higher-order thinking
- includes authentic, permissioned texts of appropriate complexity
- and documents student progress using DOK and learning progressions

Content that addresses your scope and sequence so that your assessments do not waste valuable instruction time

Help for teachers addressing the instructional shifts with content that elicits evidence of learning from each response

We constantly update our content. Ask us about what's new!

info@illuminateed.com

www.illuminateed.com

CCR Performance Tasks

Math Grade 6: Solve Problems with Fractions

Student Test Booklet

Name:

Math Grade 6: Solve Problems with Fractions

Student Rubric

This problem is meant to test if you can:
- Use a number line and an equation to represent a problem situation involving fractions;
- Generate equivalent equations;
- Solve an equation and relate the solution to a problem situation.

Your teacher will rate your answer as a level 4, 3, 2, 1, or 0. The descriptions below explain the types of answers expected at each level.

Level 4:
Your answer is correct and complete. Your answer includes:
- A number line that correctly represents the problem;
- An equation that accurately represents the situation;
- An equivalent equation and a correct explanation of why the equations are equivalent, which includes an explanation of how the operations are related;
- A correct solution of one of the equations with correct and complete work shown and a description of how the solution relates to the context of the problem.

Level 3:
Your answer is mostly correct, but one or two of your explanations are incomplete or your work contains minor errors. Your answer includes:
- A number line that correctly represents the problem;
- An equation that accurately represents the situation;
- An equivalent equation and a correct but incomplete explanation of why the equations are equivalent;
- A mostly correct solution of one of the equations, but the work may be incomplete or contain minor errors, and the description of how the solution relates to the context of the problem may be missing or incomplete.

Level 2:
You have answered only two parts, or you have some errors in several parts. Your answer may include:
- A number line that represents the problem but may contain errors such as inaccurate spacing or missing labels;
- An equation that may not accurately represent the situation;
- An incorrect or missing equivalent equation or a vague, missing, or incorrect explanation of why the equations are equivalent;
- An incorrect solution of one of the equations with work that is incomplete or contains minor errors. The description of how the solution relates to the context of the problem may be missing or incorrect.

Level 1:
Your answers are incorrect. Your answer may include:
- A number line that does not accurately represent the problem or contains significant errors;
- An equation that does not accurately represent the situation;
- An incorrect or missing equivalent equation or a vague, missing, or incorrect explanation of why the equations are equivalent;
- A solution attempt of one of the equations, with work that is incomplete or contains errors. The description of how the solution relates to the context of the problem may be missing or incorrect.

Level 0:
Your answer is not related to the question, the teacher cannot understand your answer, or you do not write anything.

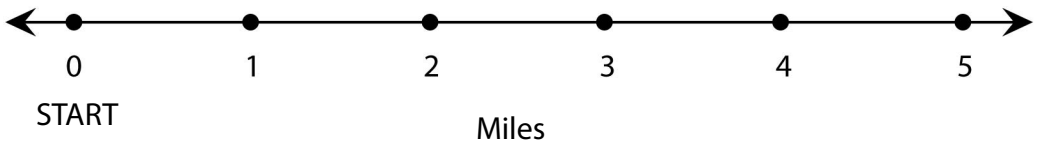
Name:_____

Math Grade 6: Solve Problems with Fractions

Complete all the tasks in the test booklet.

1 **Alex is organizing a race. The entire course is $4\frac{1}{2}$ miles long. Alex wants to put a water station every $\frac{3}{4}$ mile along the course.**

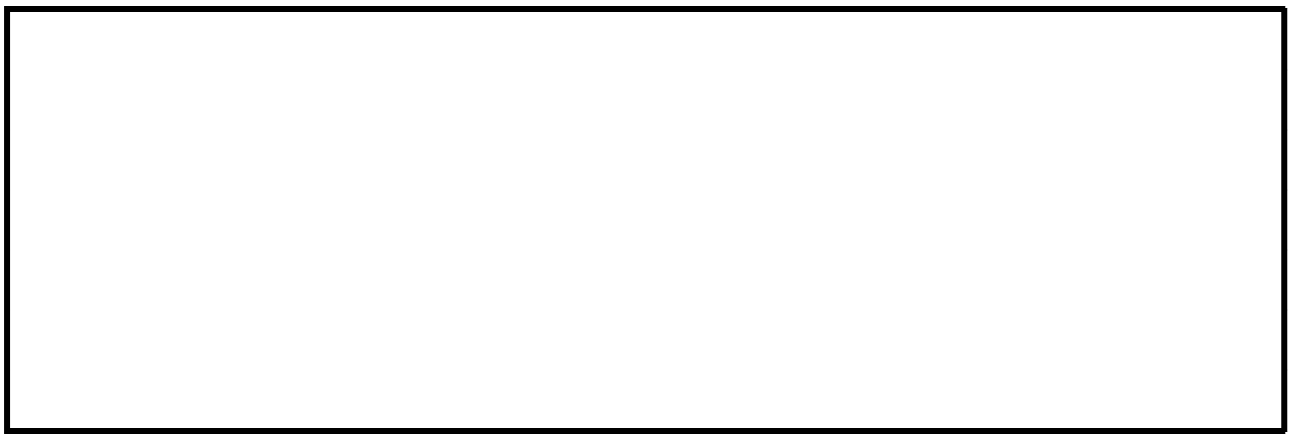
A. Use the number line below to represent the race course. The starting line is at 0 miles. Label the location of each water station and the finish line.



B. Write an equation that can be used to find the number of water stations along the entire race course. Use a variable to represent the number of water stations.

C. Write a second equation that is equivalent to the equation you wrote in part B but uses a different operation. Explain how you know that your equations are equivalent, and explain the relationship between the operations.

D. Solve one of your equations. Show your work and explain what your answer means in terms of the context of the problem about the race.



CCR Performance Tasks

Math Grade 6: Solve Problems with Fractions

Teacher Guide

About the Teacher Guide

This document contains support materials for “Math Grade 6: Solve Problems with Fractions.” This includes:

(a) The task
(b) The standards and depth of knowledge level of the task
(c) The scoring rubric
(d) Discussion questions
(e) Extension activities

These specifications have been included to help you connect the task to the Common Core content standards and the standards for mathematical practice. The rubric is designed to help you look for the development of mathematical practices in student work. It is also here to help you look for consistencies in student content errors that can help guide intervention and reteach strategies.

Test Definition File

Item #	Correct Answer	Practice Standard	Content Standards
1	See Scoring Rubric	Mathematical Practice 2	6.NS.1, 6.EE.3, 6.EE.6

SBAC Claims	PARCC Sub-Claims
1 and 2	A and D

Performance Task

Alex is organizing a race. The entire course is $4\frac{1}{2}$ miles long. Alex wants to put a water station every $\frac{3}{4}$ mile along the course.

A. Use the number line below to represent the race course. The starting line is at 0 miles. Label the location of each water station and the finish line.

B. Write an equation that can be used to find the number of water stations along the entire race course. Use a variable to represent the number of water stations.

C. Write a second equation that is equivalent to the equation you wrote in part B but uses a different operation. Explain how you know that your equations are equivalent, and explain the relationship between the operations.

D. Solve one of your equations. Show your work and explain what your answer means in terms of the context of the problem about the race.

Standards Alignment

Practice Standards

MP2 > DOK 3

Reason abstractly and quantitatively. -- Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand, considering the units involved, attending to the meaning of quantities, not just how to compute them, and knowing and flexibly using different properties of operations and objects.

Content Standards

6.NS.1

Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for* $(\frac{2}{3}) \div (\frac{3}{4})$ *and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that* $(\frac{2}{3}) \div (\frac{3}{4}) = \frac{8}{9}$ *because* $\frac{3}{4}$ of $\frac{8}{9}$ is $\frac{2}{3}$. *(In general,* $(\frac{a}{b}) \div (\frac{c}{d}) = \frac{ad}{bc}$.*) How much chocolate will each person get if 3 people share* $\frac{1}{2}$ *lb of chocolate equally? How many* $\frac{3}{4}$ *cup servings are in* $\frac{2}{3}$ *of a cup of yogurt? How wide is a rectangular strip of land with length* $\frac{3}{4}$ *mile and area* $\frac{1}{2}$ *square mile?*

6.EE.3

Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y*.

6.EE.6

Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

SBAC Claims

Mathematics Claim #1:
Concepts and Procedures. Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency.

Mathematics Claim #2:
Problem Solving. Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem solving strategies.

PARCC Sub-Claims

Sub-Claim A:
Major Content with Connections to Practices. The student solves problems involving the Major Content for her grade/course with connections to the Standards for Mathematical Practice.

Sub-Claim D:
Highlighted Practice MP.4 with Connections to Content: modeling/application. The student solves real-world problems with a degree of difficulty appropriate to the grade/course by applying knowledge and skills articulated in the standards for the current grade/course (or, for more complex problems, knowledge and skills articulated in the standards for previous grades/courses), engaging particularly in the Modeling practice, and where helpful making sense of problems and persevering to solve them (MP.1), reasoning abstractly and quantitatively (MP.2), using appropriate tools strategically (MP.5), looking for and making use of structure (MP.7), and/or looking for and expressing regularity in repeated reasoning (MP.8).

Scoring Rubric

4 Point Response:
The response demonstrates a high level of understanding. A level 4 response is characterized by:

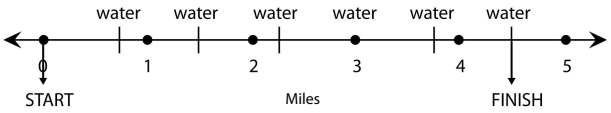
- The ability to use a visual model and an equation to represent a problem situation;
- A strong understanding of operations and equivalent equations;
- The ability to calculate with fractions and interpret the results relative to the context of a problem.

A level 4 response should include:

- A correctly labeled number line with the finish of the race labeled and marks for water stations every $\frac{3}{4}$ mile;
- A correct equation that accurately represents the situation;
- An equivalent equation and a correct and complete explanation as to why the equations are equivalent, including how the operations are related;
- Correct and complete work showing the solution of the equation with a correct interpretation of the meaning of the solution relative to the context of the problem.

A sample level 4 response follows.

Part A



Part B

$4\frac{1}{2} \div \frac{3}{4} = n$, or equivalent.

Part C

$\frac{3}{4}n = 4\frac{1}{2}$, or equivalent. "Multiplication and division are opposite operations, so multiplying by $\frac{3}{4}$ undoes dividing by $\frac{3}{4}$. To get the equivalent equation, I multiplied both sides by $\frac{3}{4}$. Since I did the same thing to both sides, the equations are equivalent."

Part D

$\frac{3}{4}n = 4\frac{1}{2}$

$\frac{4}{3} \times \frac{3}{4}n = \frac{4}{3} \times 4\frac{1}{2}$

$n = \frac{4}{3} \times \frac{9}{2} = \frac{2}{1} \times \frac{3}{1} = 6$

"The $n = 6$ means that Alex needs 6 water stations for the race. This matches the solution I found using the number line in part A."

3 Point Response:
The response demonstrates a strong understanding, but the work contains minor errors. A level 3 response is characterized by:

- The ability to use a visual model and an equation to represent a problem situation;
- A strong understanding of operations and equivalent equations, although the explanation of why the equations are equivalent may be incomplete;
- The ability to calculate with fractions and interpret the results relative to the context of a problem, although the calculations may contain minor errors.

2 Point Response:
The response demonstrates a basic but incomplete understanding of the content and a lack of ability to contextualize and decontextualize to solve a problem.
A level 2 response is characterized by:

- A basic ability to use a visual model, but the equation may not accurately represent the problem situation;
- A weak understanding of operations and equivalent equations. The response may include only one equation, or the explanation of why the equations are equivalent may be vague, incorrect, or missing;
- A basic ability to calculate with fractions but a weak ability to interpret the results relative to the context of a problem. The response likely includes a correct solution on the model in part A and a solution to an equation, but the student does not interpret the solution relative to the problem.

1 Point Response:
The response demonstrates minimal understanding. A level 1 response is characterized by:

- A weak ability to use a visual model and an equation to represent a problem situation;
- A weak understanding of operations and equivalent equations. The response may include only one equation, or the explanation of why the equations are equivalent may be vague, incorrect, or missing;
- A weak ability to calculate with fractions and interpret the results relative to the context of a problem. The response likely includes a correct solution on the model in part A, but the student may not be able to solve the equation or interpret the solution.

0 Point Response:
There is no response, or the response is off topic.

Discussion Questions

Use the following questions to help students struggling to access the problem:

1. If the race is 6 miles long and Alex places water stations every 2 miles, how many water stations does he need? How is this question similar to the one in the problem?

Possible Response: *You simply divide $\frac{6}{2} = 3$ to see that he needs three water stations. The same process is used in this problem, except that the numbers are fractions. But that doesn't change the operations used, it only changes the numbers that you use. Just as you can divide a 6-mile length into 2-mile segments, you can divide a $4\frac{1}{2}$ mile length into $\frac{3}{4}$-mile segments.*

2. How can you estimate a length of $\frac{3}{4}$ units on a number line?

Possible Response: *The length of each segment is less than 1. Further, the fraction $\frac{3}{4}$ indicates that 4 segments should have a combined total length of 3 units.*

OR

I estimated the length of $\frac{1}{4}$ and then I divided each section into quarters and then just counted by threes to make sections $\frac{3}{4}$ unit long.

3. How are multiplication and division similar? How are they different?

Possible Response: *They are inverse operations; that is, doing one operation undoes the other. For example, multiplying by 3 eliminates the result of dividing by 3. Also, these two operations can be used to generate a fact family: with multiplication, $3 \times 4 = 12$ and $4 \times 3 = 12$; with division, $12 \div 4 = 3$ and $12 \div 3 = 4$. The same rules hold for fractions; e.g., $6 \times \frac{1}{2} = 3$, so $3 \div 6 = \frac{1}{2}$.*

Extension Activities

1. Developing an understanding of why the rule of “invert and multiply” works when dividing fractions.

Students likely know that when multiplying fractions, you simply multiply the numerators and multiply the denominators. The same intuitive approach can be used for division, and doing so sheds light on why “invert and multiply” works. Do several examples where the numerator of the first fraction is divided by the numerator of the second fraction, and the denominator of the first is divided by the denominator of the second.

Sample: Divide $\frac{3}{5} \div \frac{4}{7} \rightarrow \frac{3 \div 4}{5 \div 7} \rightarrow \frac{3 \div 4}{5 \div 7} \times \frac{4}{4} \times \frac{7}{7} \rightarrow \frac{3 \times 7}{5 \times 4} = \frac{21}{20}$.

In the middle, multiply by $\frac{4}{4}$ and by $\frac{7}{7}$, both of which are equivalent to multiplying by 1. In the numerator, the division by 4 is “canceled” by the multiplication by 4; in the denominator, the division by 7 is “canceled” by the multiplication by 7. The numerator of the result is the numerator of the first fraction multiplied by the denominator of the second, and the denominator of the result is the denominator of the first fraction multiplied by the numerator of the second. That is, it’s the same result as using “invert and multiply”! In essence, “invert and multiply” is a shortcut that does what is shown here. It’s merely a shortcut for a lot of steps that we’d rather not write every time we do division with fractions.

2. Developing a deeper understanding of the connection between multiplication and division.

Develop sets of fact families with students, using both integers and fractions.

Sample: Ask students to create three equations that are equivalent to $4 \times 3 = 12$. They are likely to come up with $3 \times 4 = 12$, but they should also see that $12 \div 3 = 4$ and $12 \div 4 = 3$ are equivalent, too. Then do a similar activity using an equation involving fractions, such as $\frac{2}{3} \times \frac{3}{4} = \frac{1}{2}$.

In addition to having students generate the three equivalent equations, you may want to show diagrams that represent each situation. Then do the same activity with equations involving variables.