Inspect
CCR Performance Tasks

Math II: Model Falling Objects

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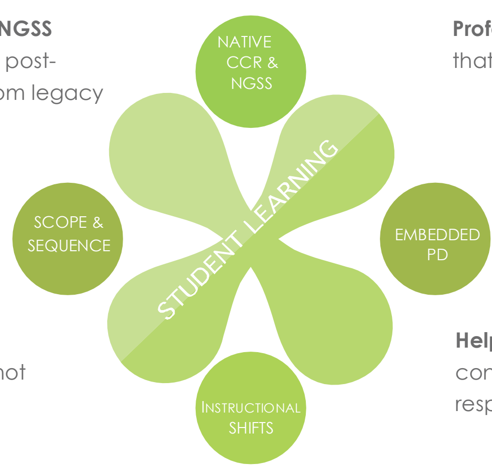

Professional development embedded within content that

- shows the relationship between specific skills and higher-order thinking
- includes authentic, permissioned texts of appropriate complexity
- and documents student progress using DOK and learning progressions

Content that addresses your scope and sequence so that your assessments do not waste valuable instruction time

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CCR Performance Tasks

Math II: Model Falling Objects

Student Test Booklet

Name:

Student Rubric

This problem tests if you can:
- Plot real-world data in order to graphically represent the relationships between variables;
- Use appropriate methods to determine an equation that describes the graph.

Your teacher will rate your answer as a level 4, 3, 2, 1, or 0. The descriptions below explain the types of answers expected at each level.

Level 4:
Your answer is correct and complete. Your answer includes:
- An accurate graph representing the data given;
- An equation that correctly describes the graph with supporting work and explanations.

Level 3:
Your answer is mostly correct but includes some minor errors. Your answer includes:
- An accurate graph representing the data given, possibly with one or two minor errors;
- Work or explanation that shows a correct strategy for determining the equation that describes the graph. The work may contain minor calculation errors that result in an incorrect equation, or the equation may be correct but the work or explanation is incomplete or missing.

Level 2:
Your answer is partially correct but is incomplete or includes some major errors. Your answer includes:
- A graph representing the data given, possibly with several errors;
- An attempt to determine the equation that describes the graph, but the work is incomplete and does not include a final equation or includes significant errors.

Level 1:
Your answer is incorrect. Your answer includes:
- An incorrect graph that fails to represent the data given;
- An incorrect function that fails to represent the graph of the data.

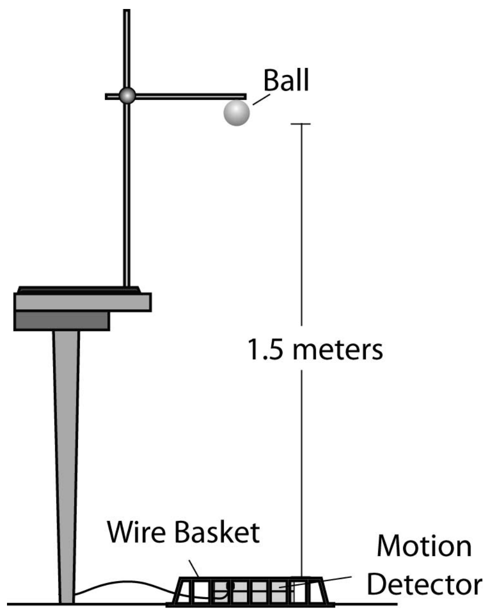
Level 0:
Your answer is not related to the question, the teacher cannot understand your answer, or you do not write anything.

Name:_______________________________

Math II: Model Falling Objects

Complete all the tasks in the test booklet.

1 **As part of a project for their physics class, some students are determining functions that express the distance-time relationships of objects in motion. The students use a motion detector to measure the exact height of a ball at regular intervals. The figure below indicates the materials used in the experiment and how they are set up.**



A ball is positioned directly over the motion detector, which is protected by a wire basket. The ball is dropped from a height of 1.5 meters. As soon as the ball hits the wire basket, the motion detector starts to measure the height of the ball at 0.10 second intervals. It stops measuring the height of the ball as soon as the ball bounces on the wire basket the second time. The data from the experiment are shown in the table.

Go On

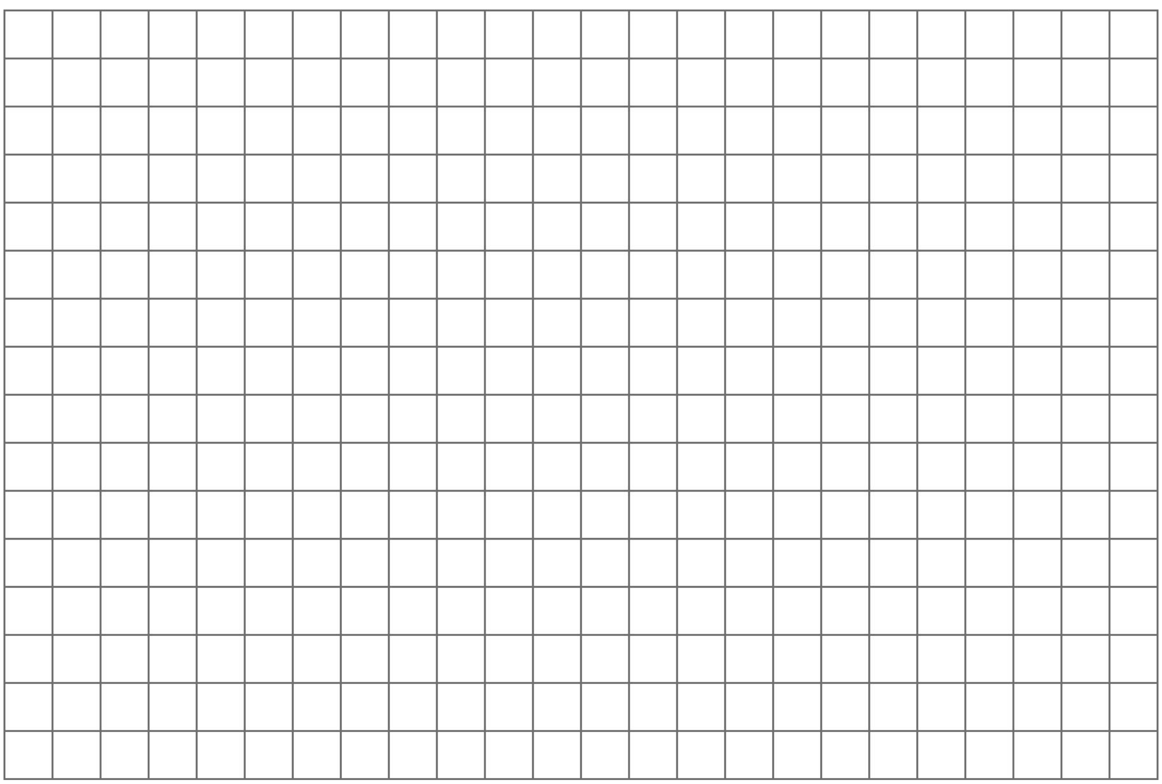
Name:________________________________

Math II: Model Falling Objects

Height from First Bounce to Second Bounce

Time (seconds)	Height (meters)
0.00	0.00
0.10	0.45
0.20	0.80
0.30	1.05
0.40	1.20
0.50	1.25
0.60	1.20
0.70	1.05
0.80	0.80
0.90	0.45
1.00	0.00

A. Graph this data set on the grid. (Hint: graph the height along the *y*-axis and the time along the *x*-axis).



Go On

Name:________________________________

B. Determine an equation that describes the height (*h*) of the ball as a function of time (*t*) and matches the graph. You can determine the equation algebraically or use a graphing application. Show your work or explain how you determined the equation. Use screen shots or sketches to illustrate your explanation of how you used the graphing application to determine the equation.

STOP

CCR Performance Tasks

Math II: Model Falling Objects

Teacher Guide

About the Teacher Guide

This document contains support materials for “Math II: Model Falling Objects.”
This includes:

(a) The task
(b) The standards and depth of knowledge level of the task
(c) The scoring rubric
(d) Discussion questions
(e) Extension activities

These specifications have been included to help you connect the task to the Common Core content standards and the standards for mathematical practice. The rubric is designed to help you look for the development of mathematical practices in student work. It is also here to help you look for consistencies in student content errors that can help guide intervention and reteach strategies.

Test Definition File

Item #	Correct Answer	Practice Standard	Content Standards
1	See Scoring Rubric	Mathematical Practice 4	F-BF.1.a; F-BF.3

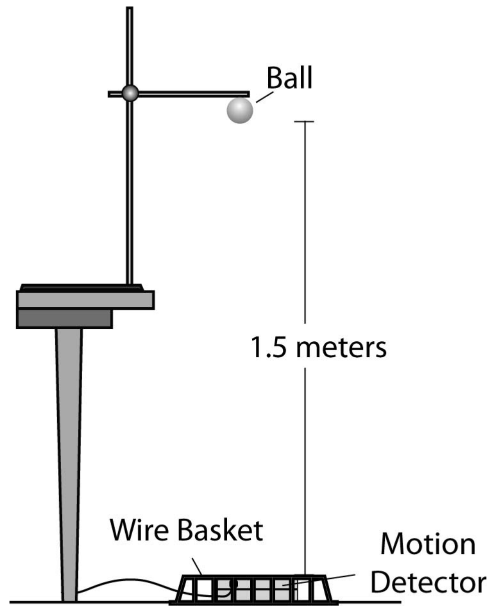
SBAC Claims	PARCC Sub-Claims
1 and 4	A and D

Special Instructions

Consider allowing students to use a graphing application to determine the equation of the function for part B. A free graphing application can be downloaded from http://www.padowan.dk/download/. Tell students which types of technology (calculator or computer) they can use for part B, if any.

Performance Task

As part of a project for their physics class, some students are determining functions that express the distance-time relationships of objects in motion. The students use a motion detector to measure the exact height of a ball at regular intervals. The figure below indicates the materials used in the experiment and how they are set up.



A ball is positioned directly over the motion detector, which is protected by a wire basket. The ball is dropped from a height of 1.5 meters. As soon as the ball hits the wire basket, the motion detector starts to measure the height of the ball at 0.10 second intervals. It stops measuring the height of the ball as soon as the ball bounces on the wire basket the second time. The data from the experiment are shown in the table.

Height from First Bounce to Second Bounce

Time (seconds)	Height (meters)
0.00	0.00
0.10	0.45
0.20	0.80
0.30	1.05
0.40	1.20
0.50	1.25
0.60	1.20
0.70	1.05
0.80	0.80
0.90	0.45
1.00	0.00

A. Graph this data set on the grid. (Hint: graph the height along the *y*-axis and the time along the *x*-axis).

B. Determine an equation that describes the height (*h*) of the ball as a function of time (*t*) and matches the graph. You can determine the equation algebraically or use a graphing application. Show your work or explain how you determined the equation. Use screen shots or sketches to illustrate your explanation of how you used the graphing application to determine the equation.

Standards Alignment

Practice Standards

MP4 > DOK 3

Model with mathematics. -- Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Content Standards

F-BF.1
Write a function that describes a relationship between two quantities.
a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

F-BF.3
Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*

SBAC Claims

Mathematics Claim #1:
Concepts and Procedures. Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency.

Mathematics Claim #4:
Modeling and Data Analysis. Students can analyze complex, real-world scenarios and can use mathematical models to interpret and solve problems.

PARCC Sub-Claims

Sub Claim A:
Major Content with Connections to Practices. The student solves problems involving the Major Content for her grade/course with connections to the Standards for Mathematical Practice.

Sub Claim D:
Highlighted Practice MP.4 with Connections to Content: modeling/application. The student solves real-world problems with a degree of difficulty appropriate to the grade/course by applying knowledge and skills articulated in the standards for the current grade/course (or, for more complex problems, knowledge and skills articulated in the standards for previous grades/courses), engaging particularly in the Modeling practice, and where helpful making sense of problems and persevering to solve them (MP.1), reasoning abstractly and quantitatively (MP.2), using appropriate tools strategically (MP.5), looking for and making use of structure (MP.7), and/or looking for and expressing regularity in repeated reasoning (MP.8).

Scoring Rubric

4 Point Response:

The response demonstrates a high level of understanding, including:

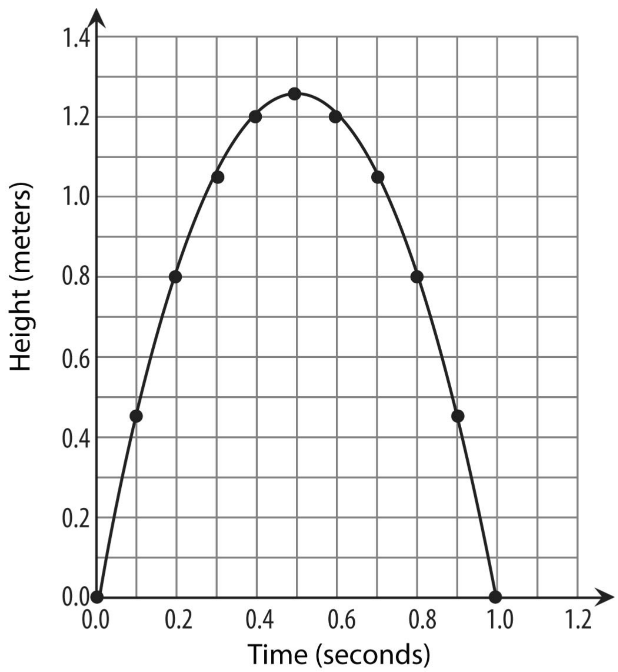
- A strong ability to correctly and accurately plot real-world data;
- A strong ability to determine an equation that describes the graph of the data.

A level 4 response is characterized by:

- An accurate graph representing the data given in the table;
- A correctly formed function that matches the graph of the data, with supporting work, explanation, and screen shots or sketches.

A sample level 4 response follows.

Part A:

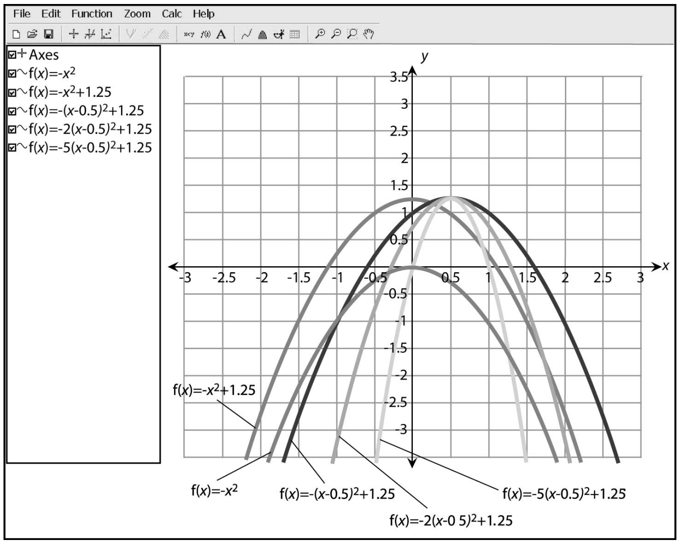


Part B (sample algebraic solution):

"This graph has the shape of a downward-facing parabola, so it is a quadratic. The roots (where $h = 0$) are $t = 0$ and $t = 1.0$, so I know that $t = 0$ and $t - 1 = 0$. But many parabolas have those roots so I still don't know the exact equation. I can instead use the general form $y = ax^2 + bx + c$ and substitute in three points to get three equations. For (0,0), $0 + 0 + c = 0$, so $c = 0$. For (1,0), $0 = a + b$, so $a = -b$. For (0.5,1.25) (the vertex), $1.25 = a(0.5)^2 - a(0.5) \rightarrow 1.25 = a(0.25) - a(0.5) \rightarrow 1.25 = -a(0.25) \rightarrow a = -5$. So the equation of the parabola is $h(t) = -5t^2 + 5t$. The initial position (height) was 0, which is why c from the general quadratic form is 0."

Part B (sample determining the function using a graphing application):

"It looks like an upside down parabola so I started by graphing $f(x) = -x^2$. You can see this in the screen shot as the curve with the vertex at the origin. I know the height of the vertex needs to be at 1.25 instead of 0, so I graphed $f(x) = -x^2 + 1.25$ next. You can see that in my screen shot as the curve with the vertex at (0, 1.25). Then I saw that I need to move the parabola to the right 0.5 units so I graphed $f(x) = -(x - 0.5)^2 + 1.25$. This is the widest curve with vertex (0.5, 1.25) in my screen shot. Then I saw that the curve is too wide; it should cross the x-axis at 0 and 1. So I tried graphing $f(x) = -2(x - 0.5)^2 + 1.25$. This is the second widest curve in my screen shot. But it was still too wide, so I tried graphing $f(x) = -5(x - 0.5)^2 + 1.25$. This is the correct function. The vertex is at (0.5, 1.25) and the x-intercepts are (0,0) and (1,0)."



3 Point Response:

The response demonstrates a strong understanding, but the work contains minor errors. A level 3 response is characterized by:

- A strong ability to correctly and accurately plot real-world data;
- A strong ability to determine an equation that describes the graph, although the work or explanations may be incomplete or include one or two minor errors.

2 Point Response:

The response demonstrates a basic but incomplete understanding. A level 2 response is characterized by:

- A strong ability to plot real-world data, although the graph may include several minor errors;
- A weak to basic understanding of how to determine an equation that describes the graph. The work and explanation demonstrate an attempt to determine the equation but are incomplete or result in an incorrect equation.

1 Point Response:

The response demonstrates minimal understanding. A level 1 response is characterized by:

- A weak ability to plot real-world data, possibly with errors severe enough to pose a barrier to successful completion of the task;
- An inability to determine an equation that describes the given experimental data.

0 Point Response:

There is no response, or the response is off topic.

Discussion Questions

Use the following questions to help students struggling to access the problem:

1. What does the graph of a quadratic function look like?

Possible response: *A quadratic function, when graphed, takes the shape of a parabola. A parabola has a vertex and is symmetrical about a line that is parallel to the* y*-axis and whose vertex lies on that line.*

2. What are the different methods of finding the equation of a parabola on a graph? Why are some methods more appropriate for certain parabolas but not for others?

Possible response: *You can find the roots but then you need to substitute the x and y values for some points into the general form $y= ax^2 + bx + c$ to determine the values of a, b, and c. Another method is to use the vertex form of the general equations, $y = a(x - h)^2 + k$, where the ordered pair (h, k) is the coordinates of the vertex of the parabola.*

3. Once you have found an equation for a function that you think matches the experimental data, how can you check that the function is a good model?

Possible response: *You can plug numbers from the experimental data into the equation to see if they satisfy the function. Since the function also represents values on a graph, you can graph the function using a graphing calculator and plot the points from the experimental data. If the points coincide, then the function is a good model.*

Extension Activities

1. Make a graph and write an equation to describe a different part of the ball's trajectory.

   A. The students run the experiment again, but this time they measure the distance-time relationship from when the ball is first dropped to the time that it first bounces on the wire basket. The table shows that the ball starts at a height of 1.50 meters above the wire basket. After 0.10 seconds, the ball is at a height of 1.45 meters above the basket, and so on.

Height from Initial Drop Until the First Bounce

Time (seconds)	Height (meters)
0.00	1.50
0.10	1.45
0.20	1.30
0.30	1.05
0.40	0.70
0.50	0.25
0.55	0.00

   B. Write a function $h(t)$ that matches this graph. Which initial value(s) should be equal to zero for this function? Show your work and explain your reasoning.

2. How would the same experiments work on the Moon?

   A. The functions you developed model the relationships between distance, velocity, and acceleration for the moving ball in the experiments. The ball is under the constant influence of gravity, which causes it to accelerate downward even when it is moving in the upward direction. The acceleration due to gravity near Earth's surface is approximately 10 meters per second^2. The acceleration due to gravity on the Moon is much less — about 1.6 meters per second^2. Use the functions you developed in parts A and B and extension activity 1 to write an equation that expresses height as a function of time for a ball dropped from a height of 1.5 meters on the Moon. Show your work and explain your reasoning.

   B. In what way(s) do you expect the shapes of the curves you graph to be the same, and in what way(s) do you expect them to be different? Explain.

3. Develop a further understanding of quadratic equations and functions.

Given the generalized equation for the position of an object under constant acceleration, position = initial position + initial velocity (t) + acceleration (t)^2, explore how the graph of this function changes when initial position, initial velocity, or acceleration change. (Real-world application of F-IF.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph polynomial functions, identifying zeros when suitable factorizations are available and showing end behavior.)

4. Model physical phenomena with quadratic functions. This activity is DOK 4 and connects to science curriculum.

Model successive bounces of the ball. If appropriate equipment is available (check with your science department; many physics classrooms have motion detectors and software for data collection and analysis), have students repeat the experiment described, but collect data after multiple bounces of the ball. If such equipment is not available, present the students with data for a bouncing ball for which the acceleration stays constant but the initial velocity at the beginning of the bounce decreases each time the ball bounces. This results in a successive set of parabolas with decreasing heights at the vertex but with a constant coefficient of the t^2 term. Have students write functions that model each successive bounce and relate the differences between the functions to the physical variables of distance, velocity, and acceleration. The discussion may extend to a discussion of energy, momentum, and elastic and inelastic collisions.